

# Mathematical Proofs

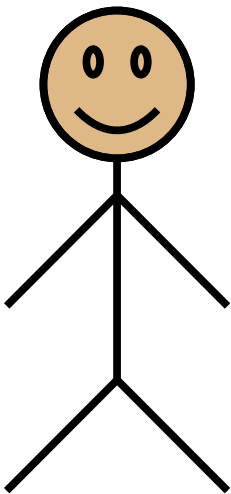
# Outline for Today

- ***How to Write a Proof***
  - Synthesizing definitions, intuitions, and conventions.
- ***Proofs on Numbers***
  - Working with odd and even numbers.
- ***Universal and Existential Statements***
  - Two important classes of statements.
- ***Variable Ownership***
  - Who owns what?

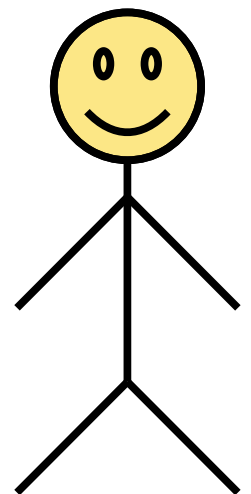
What is a Proof?

# Proof as Dialog

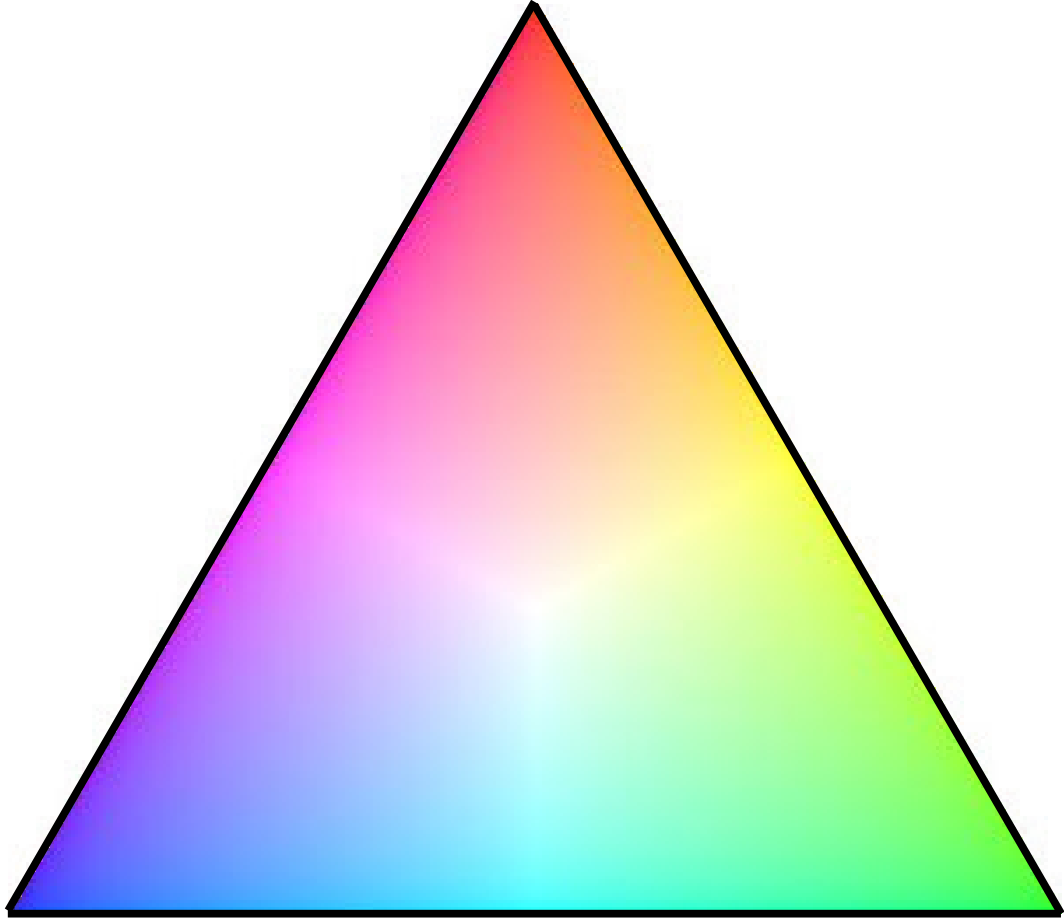
- A mathematical proof is a dialog between two parties: a ***proof writer*** and a ***proof reader***.
  - The ***proof writer*** knows a mathematical fact.
  - The ***proof reader*** is honest but skeptical.
- The proof writer's job is to take the reader on a journey from ignorance to understanding.



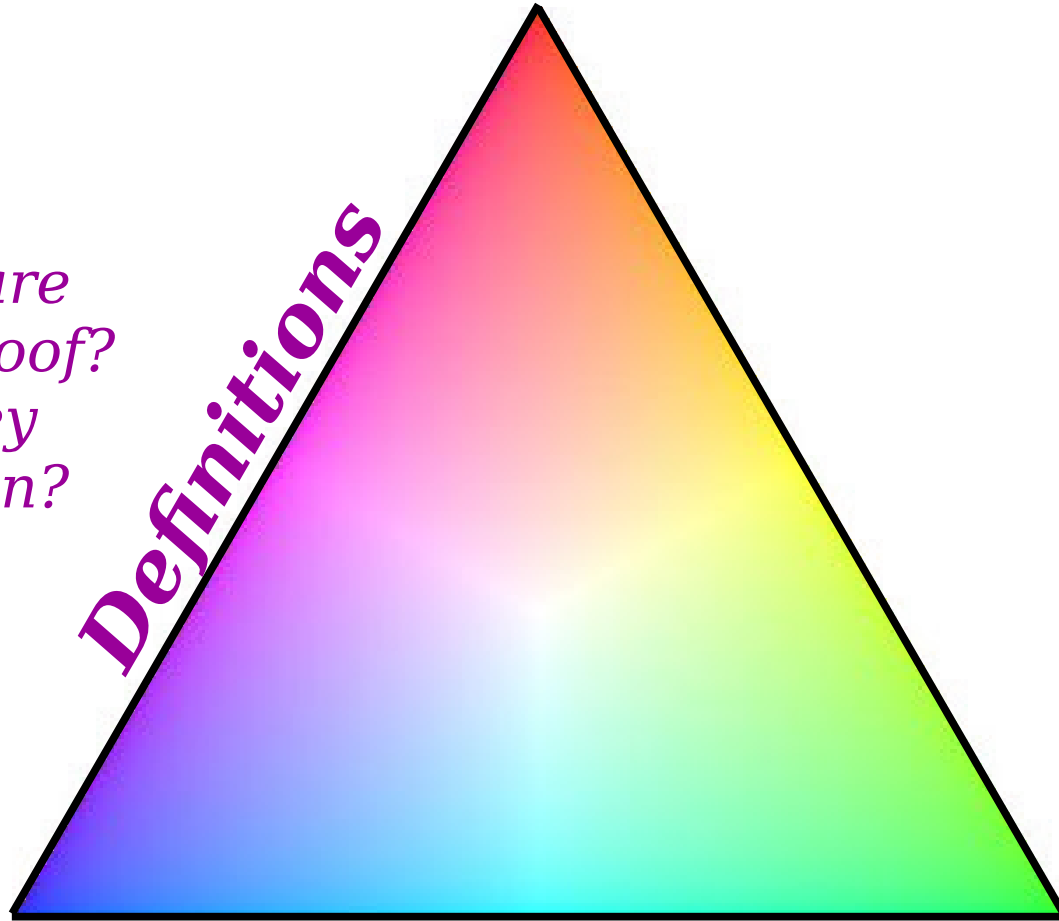
***Proof Writer (You)***



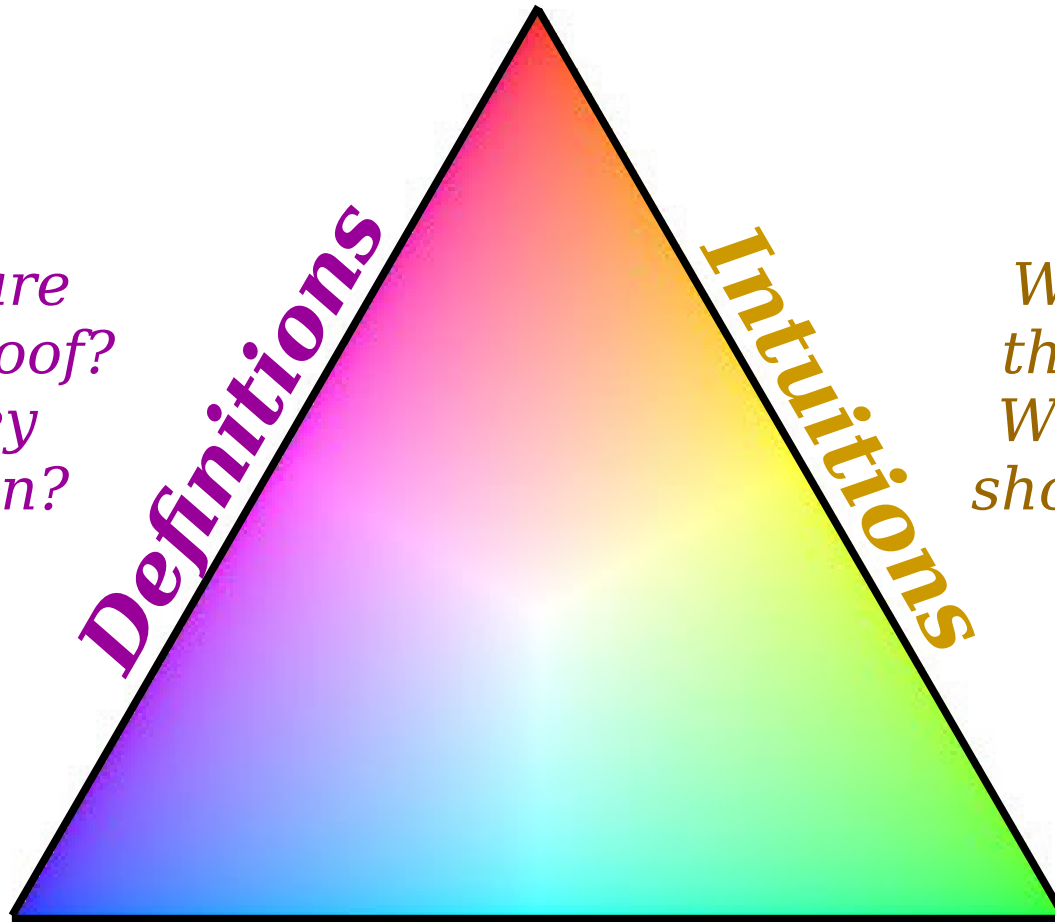
***Proof Reader***



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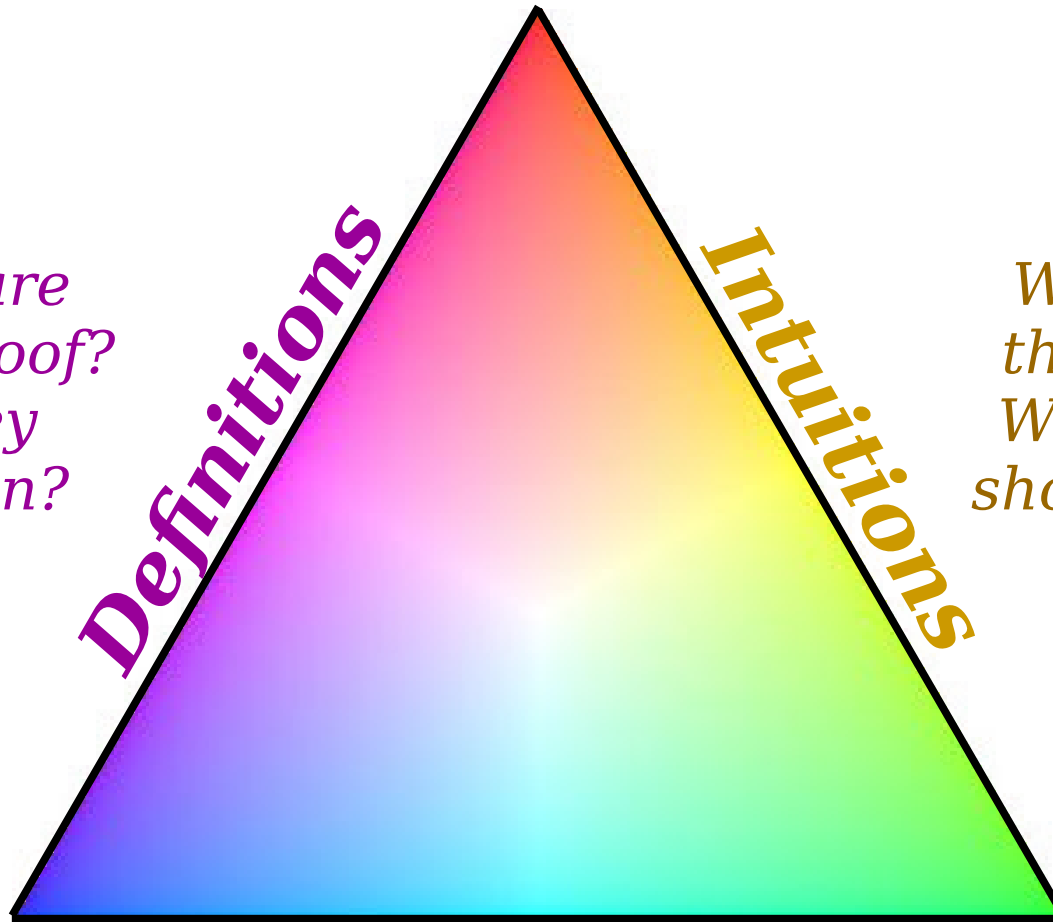
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***Conventions***

*What is the standard  
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What are the techniques  
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# Writing our First Proof

***Theorem:*** If  $n$  is an even integer,  
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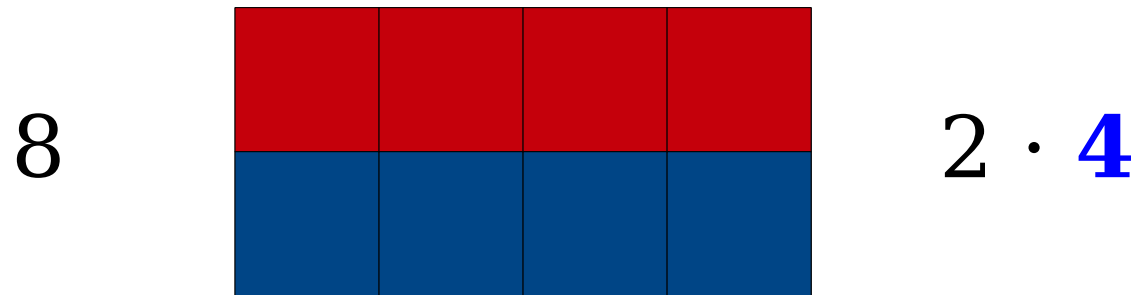
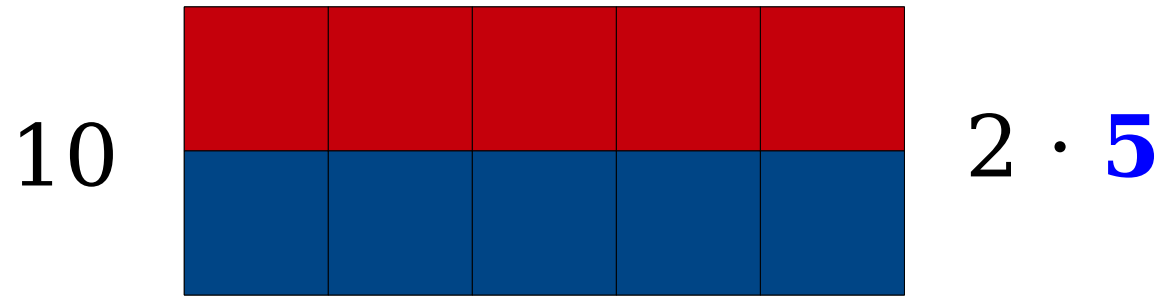
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***Theorem:*** If  $n$  is an **even** integer,  
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An integer  $n$  is called ***even*** if there is an integer  $k$  where  $n = 2k$ .

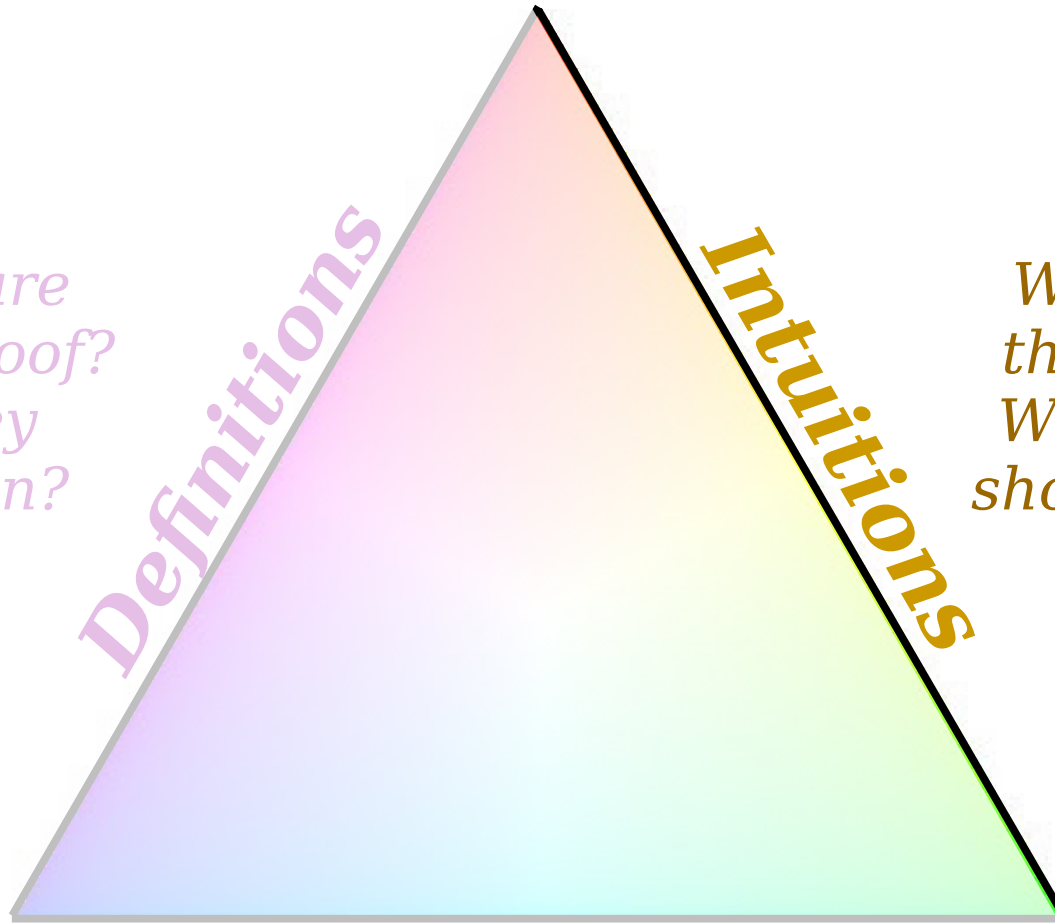
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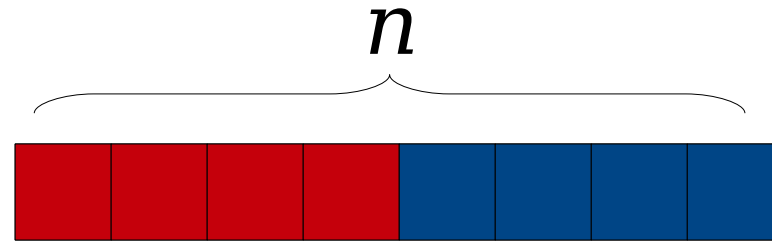
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# Let's Try Some Examples!

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# Let's Draw Some Pictures!

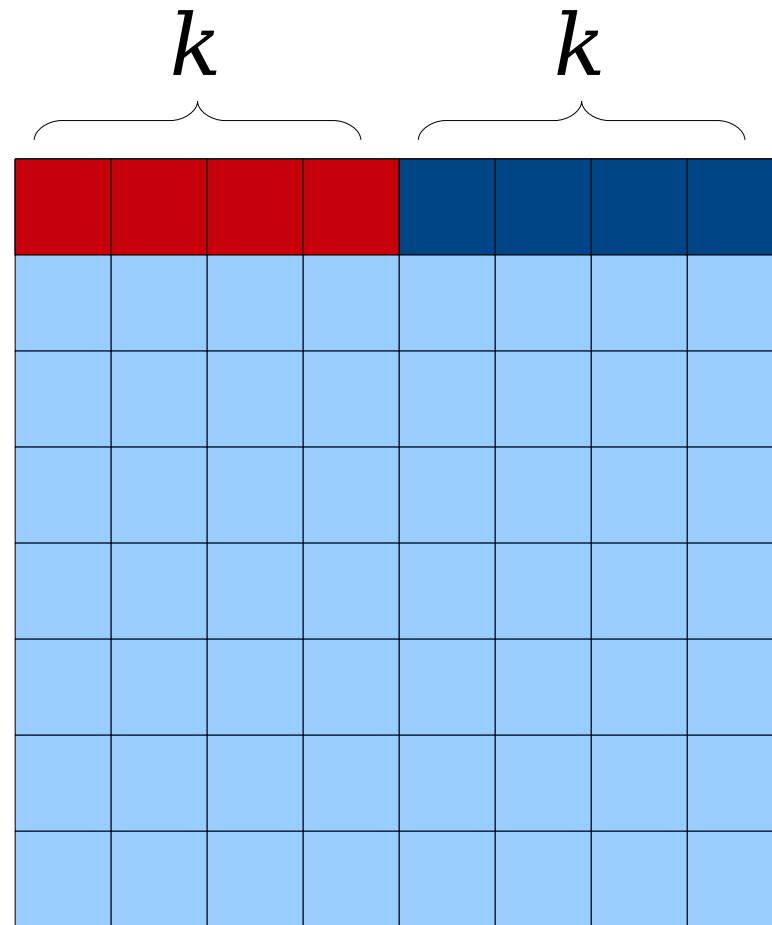


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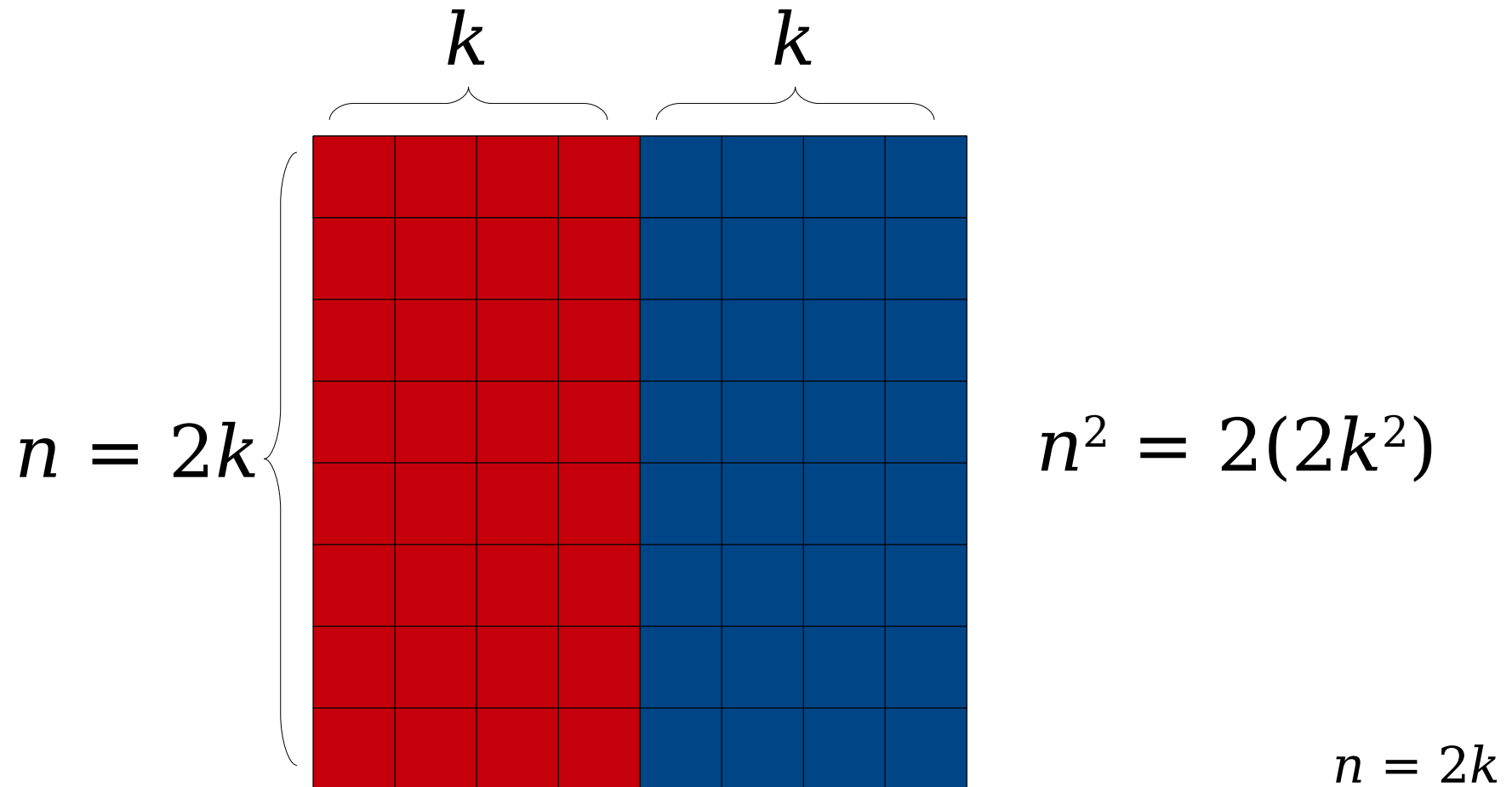
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$$n^2 = (2k)^2$$

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This symbol  
means "end of  
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To prove a statement of the form

**“If  $P$  is true, then  $Q$  is true,”**

start by asking the reader to assume that  **$P$**  is true.

From this, we see that  $n^2 = (2k)^2 = 4k^2$  (namely,  $2k^2$ ) which is even, which is

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To prove a statement of the form

**“If  $P$  is true, then  $Q$  is true,”**

From this, we assume  $P$  is true, then need to show that  $Q$  is true. Here, we're telling the reader where we're headed.

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**Proof:** Assume  $n$  is an even integer. We want to show that  $n^2$  is even.

Since  $n$  is even, there is some integer  $k$  such that  $n = 2k$ . This means that

This is the definition of an even integer. We need to use this definition to make this proof rigorous.

From this, we can see that  $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$  (namely,  $2k^2$ ) where  $n^2 = 2m$ . Therefore,  $n^2$  is even, which is what we wanted to show. ■

# Our First Proof!

**Theorem:** If

**Proof:** Assume  
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Notice how we use the value of  $k$  that we obtained above. Giving names to quantities, allows us to manipulate them. This is similar to variables in programs.

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**Proof:** Assume  $n$  is an even integer. We want to show that  $n^2$  is even.

Since  
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Our ultimate goal is to prove that  $n^2$  is even. This means that we need to find some  $m$  where

$n^2 = 2m$ . Here, we're explicitly showing how we can do that.

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Hey, that's what we said we were going to do! We're done.

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Our Next Proof

***Theorem:*** For any integers  $m$  and  $n$ ,  
if  $m$  and  $n$  are odd, then  $m + n$  is even.

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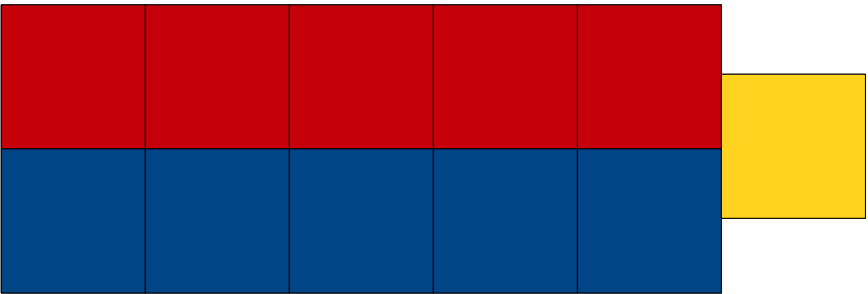
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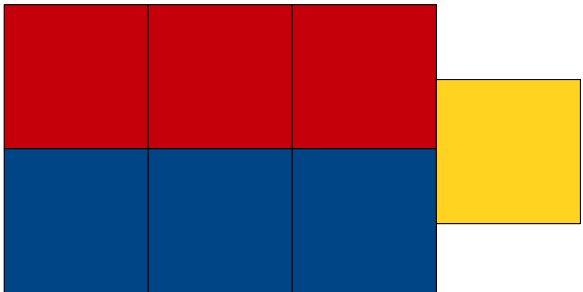
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***Theorem:*** For any integers  $m$  and  $n$ , if  $m$  and  $n$  are **odd**, then  $m + n$  is even.

11   $2 \cdot 5 + 1$

7   $2 \cdot 3 + 1$

1   $2 \cdot 0 + 1$

---

An integer  $n$  is called **odd** if there is an integer  $k$  where  $n = 2k + 1$ .

Going forward, we'll assume the following:

1. Every integer is either even or odd.
2. No integer is both even and odd.

***Theorem:*** For any integers  $m$  and  $n$ , if  $m$  and  $n$  are odd, then  $m + n$  is even.

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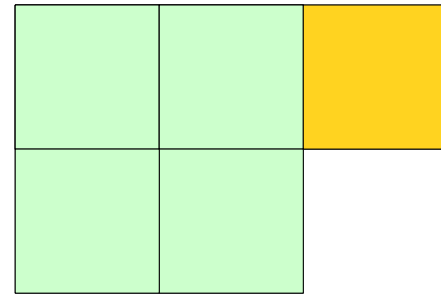
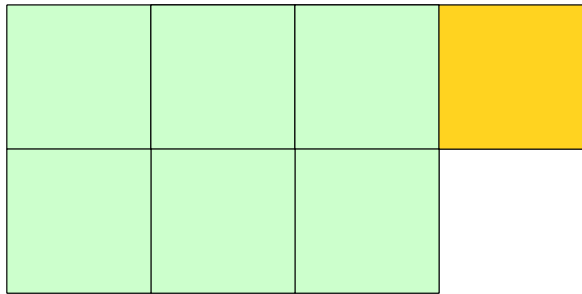
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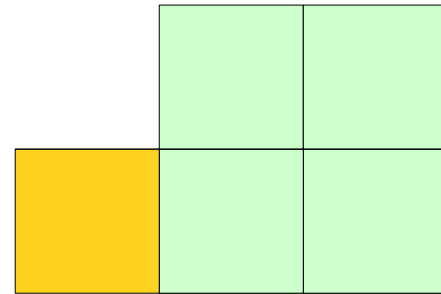
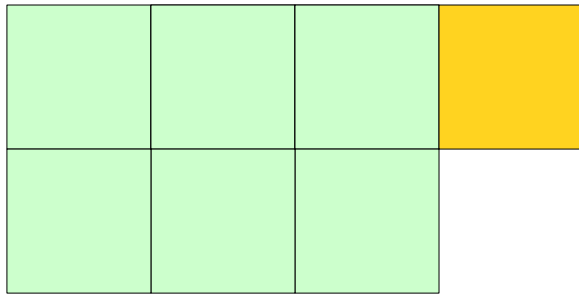
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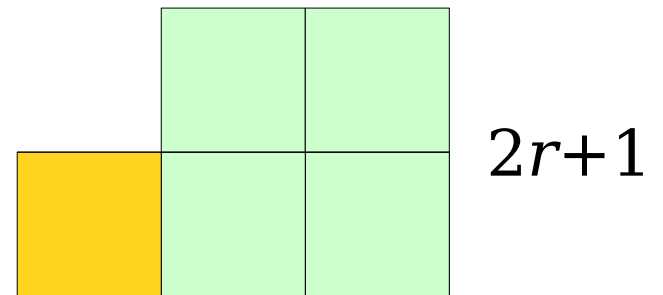
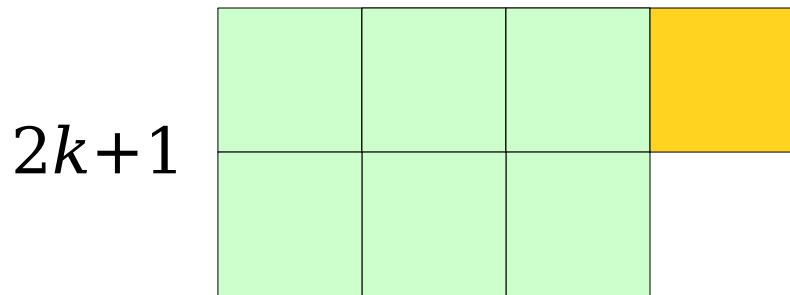
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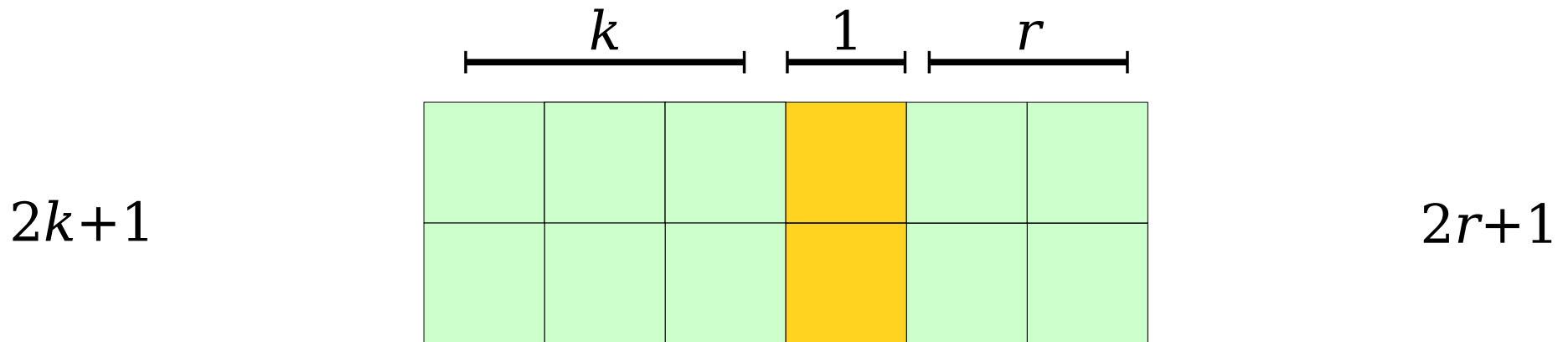
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$$(2k+1) + (2r+1) = 2(k + r + 1)$$

---

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Similarly, because  $n$  is odd there must be some integer  $r$  such that

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By adding equations (1) and (2) we learn that

$$m + n = 2k + 1 + 2r + 1$$

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By letting the reader pick  $m$  and  $n$  arbitrarily, anything we prove about  $m$  and  $n$  will generalize to all possible choices for those values.

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Since  $m$  is

To prove a statement of the form

**“If  $P$  is true, then  $Q$  is true,”**

Similarly, b

n that

By adding

start by asking the reader to assume that  **$P$**  is true.

$$= 2k + 2r + 2$$

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**Proof:** Consider any odd. We need to show that  $m + n$  is even. Since  $m$  is odd, we can write

Numbering these equalities lets us refer back to them later on, making the flow of the proof a bit easier to understand.

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This is a complete sentence! Proofs are expected to be written in complete sentences, so you'll often use punctuation at the end of formulas.

We recommend using the "mugga mugga" test - if you read a proof and replace all the mathematical notation with "mugga mugga," what comes back should be a valid sentence.

learn that

$$m + n = 2k + 1 + 2r + 1$$

$$= 2k + 2r + 2$$

$$= 2(k + r + 1). \quad (3)$$

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# Some Little Exercises

- Here's a list of other theorems that are true about odd and even numbers:
  - **Theorem:** The sum and difference of any two even numbers is even.
  - **Theorem:** The sum and difference of an odd number and an even number is odd.
  - **Theorem:** The product of any integer and an even number is even.
  - **Theorem:** The product of any two odd numbers is odd.
- Going forward, we'll just take these results for granted. Feel free to use them in the problem sets.
- If you'd like to practice the techniques from today, try your hand at proving these results!

# Universal and Existential Statements

***Theorem:*** For any odd integer  $n$ ,  
there exist integers  $r$  and  $s$  where  $r^2 - s^2 = n$ .

*What terms are used in this proof?  
What do they formally mean?*

***Definitions***

***Intuitions***

*What does this theorem mean?  
Why, intuitively, should it be true?*

***Conventions***

*What is the standard format for writing a proof?  
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there exist integers  $r$  and  $s$  where  $r^2 - s^2 = n$ .

This result is true for every possible choice of odd integer  $n$ . It'll work for  $n = 1$ ,  $n = 137$ ,  $n = 103$ , etc.

***Theorem:*** For any odd integer  $n$ ,  
there exist integers  $r$  and  $s$  where  $r^2 - s^2 = n$ .

We aren't saying this is true for every choice of  $r$  and  $s$ . Rather, we're saying that *somewhere out there* are choices of  $r$  and  $s$  where this works.

# Universal vs. Existential Statements

- A ***universally-quantified statement*** is a statement of the form

**For all  $x$ , [some-property] holds for  $x$ .**

- We've seen how to prove these statements.
- An ***existentially-quantified statement*** is a statement of the form

**There is some  $x$  where [some-property] holds for  $x$ .**

- How do you prove an existentially-quantified statement?

# Proving an Existential Statement

- Over the course of the quarter, we will see several different ways to prove an existentially-quantified statement of the form  
**There is an  $x$  where [some-property] holds for  $x$ .**
- ***Simplest approach:*** Search far and wide, find an  $x$  that has the right property, then show why your choice is correct.

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What do they  
formally mean?*

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# Let's Try Some Examples!

$$1 = \underline{\quad}^2 - \underline{\quad}^2$$

$$3 = \underline{\quad}^2 - \underline{\quad}^2$$

$$5 = \underline{\quad}^2 - \underline{\quad}^2$$

$$7 = \underline{\quad}^2 - \underline{\quad}^2$$

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***Theorem:*** For any odd integer  $n$ , there exist integers  $r$  and  $s$  where  $r^2 - s^2 = n$ .

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***Educated Guess:***

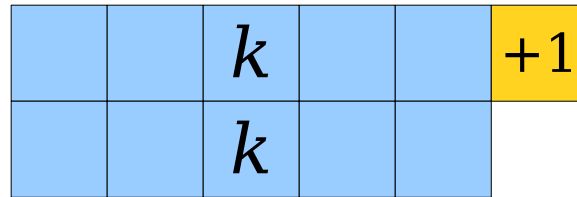
$$2k + 1 = (k+1)^2 - k^2.$$

$$\mathbf{3} + 1 = \mathbf{4}^2 - \mathbf{3}^2$$

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***Theorem:*** For any odd integer  $n$ ,  
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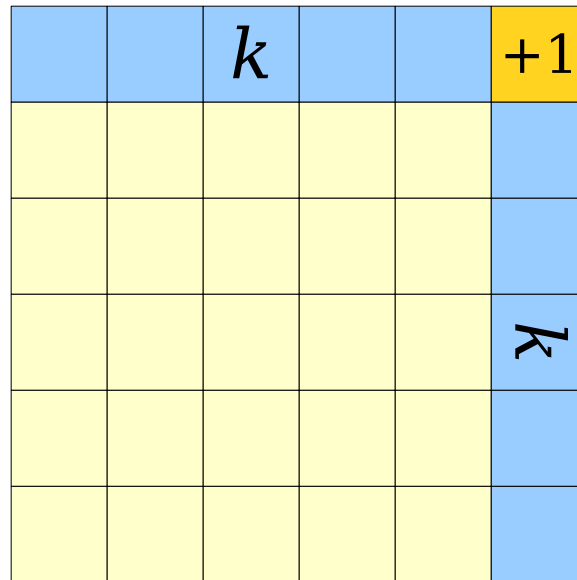
# Let's Draw Some Pictures!



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# Let's Draw Some Pictures!



$$(k+1)^2 - k^2 = 2k+1$$

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$$\begin{aligned} r^2 - s^2 &= (k+1)^2 - k^2 \\ &= k^2 + 2k + 1 - k^2 \\ &= 2k + 1 \\ &= n. \end{aligned}$$

As always, it's helpful to write out what we need to demonstrate with the rest of the proof.

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We're trying to prove an existential statement. The easiest way to do that is to just give concrete choices of the objects being sought out.

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Check the appendix to this slide deck for more about who gets to choose values.

**Time-Out for Announcements!**

# Working in Pairs

- Problem Set Zero is due this Monday at 1:00PM. It must be completed individually.
- After that, the remaining problem sets can be done individually or in pairs.
  - Each pair should make a single joint submission.
- We have advice about how to work effectively in pairs up on the course website – check the “Guide to Partners.”
- Want to work in a pair, but don’t know who to work with? Fill out [\*\*this Google form\*\*](#) and we’ll connect you with a partner on Monday.

# Problem Set One

- Problem Set One goes out today. It's due next Friday at 1:00PM.
  - Explore the language of set theory and better intuit how it works.
  - Learn more about the structure of mathematical proofs.
  - Write your first “freehand” proofs based on your experiences.
- As always, reach out if you have any questions!

# Lecture 02

- Because our first lecture this quarter was on Wednesday, there's one additional lecture we'll need to cover before the start of next week.
- That lecture is prerecorded and is available on Canvas.
- It's important to watch this lecture before Monday's lecture and before starting Problem Set 1.
- Please feel free to post questions on EdStem!

# Office Hours

- It is ***completely normal*** in this class to need to get help from time to time.
- Feel free to ask clarifying and conceptual questions on EdStem.
- Need more structured help? We have office hours! Feel free to stop on by.
  - Check out the online “Guide to Office Hours” for more information about how our office hours system works.
  - The OH calendar is available on the course website.
- Office hours start this Monday.

Back to CS103!

***Theorem:*** If  $n$  is an integer,  
then  $\lceil n/2 \rceil + \lfloor n/2 \rfloor = n$ .

*What terms are used in this proof?  
What do they formally mean?*

***Definitions***

***Intuitions***

*What does this theorem mean?  
Why, intuitively, should it be true?*

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# Floors and Ceilings

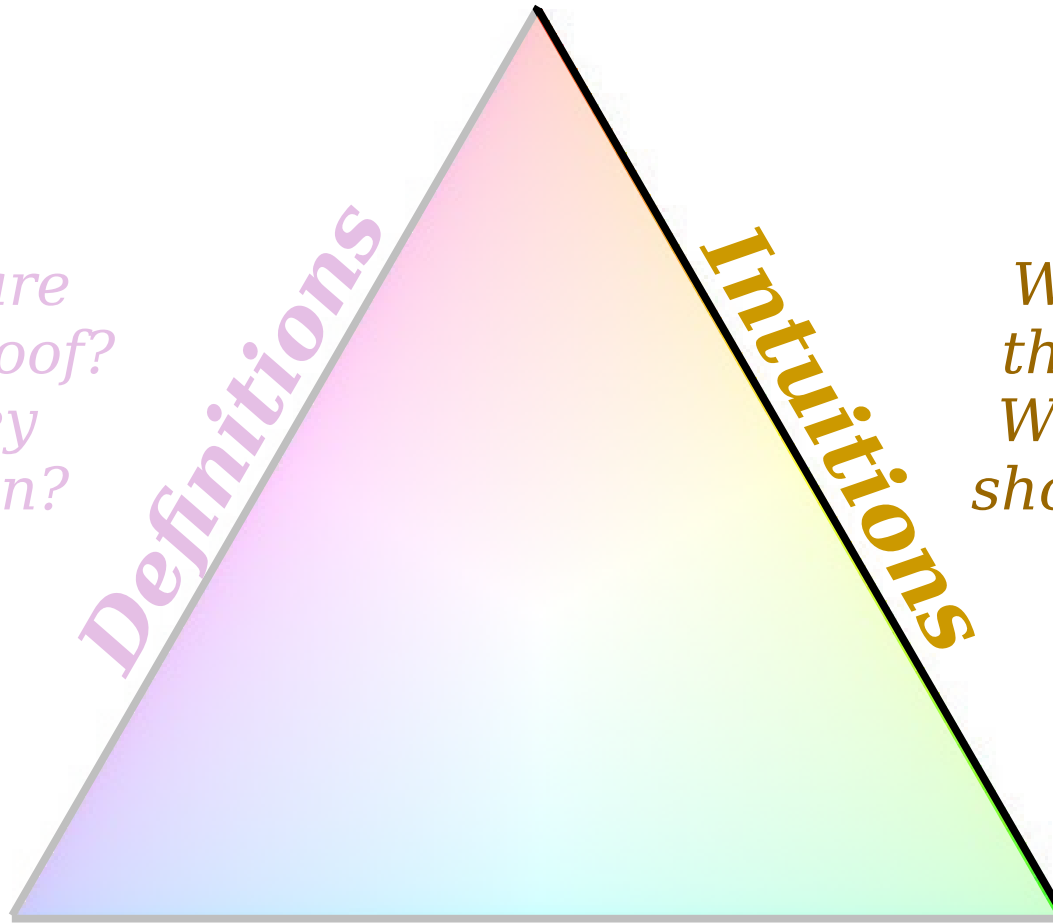
- The notation  $\lceil x \rceil$  represents the **ceiling** of  $x$ , the smallest integer greater than or equal to  $x$ .
  - What is  $\lceil 1 \rceil$ ? What's  $\lceil 1.2 \rceil$ ? What's  $\lceil -1.2 \rceil$ ?
  - **Intuition:** Start at  $x$  on the number line, then move to the right until you hit a tick mark.
- The notation  $\lfloor x \rfloor$  represents is the **floor** of  $x$ , the largest integer less than or equal to  $x$ .
  - What is  $\lfloor 1 \rfloor$ ? What's  $\lfloor 1.2 \rfloor$ ? What's  $\lfloor -1.2 \rfloor$ ?
  - **Intuition:** Start at  $x$  on the number line, then move to the left until you hit a tick mark.

*What terms are used in this proof?  
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***Definitions***

*What does this theorem mean?  
Why, intuitively, should it be true?*

***Intuitions***



***Conventions***

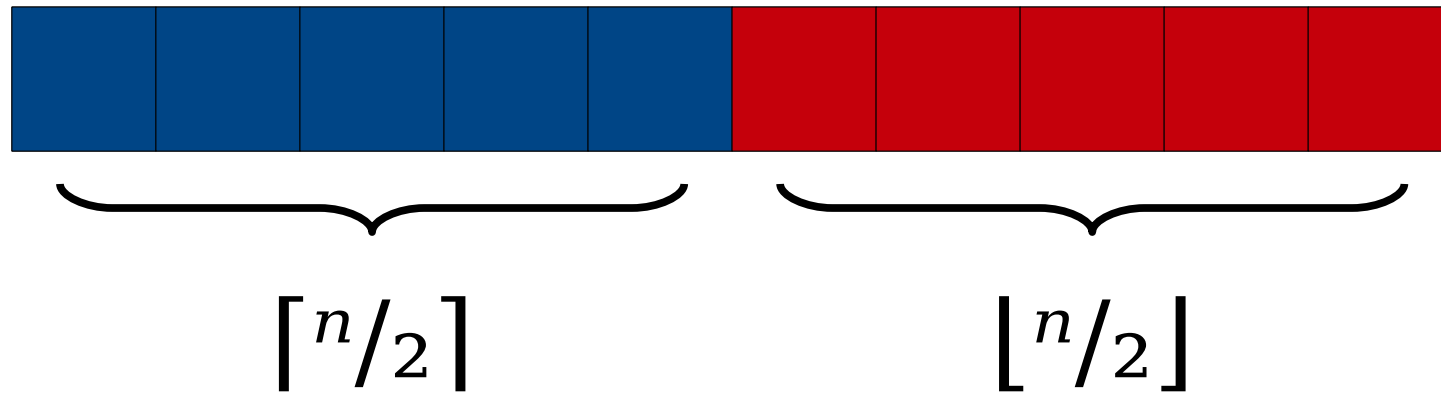
*What is the standard format for writing a proof?  
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# Let's Try Some Examples!

---

***Theorem:*** If  $n$  is an integer, then  $\lceil n/2 \rceil + \lfloor n/2 \rfloor = n$ .

# Let's Draw Some Pictures!

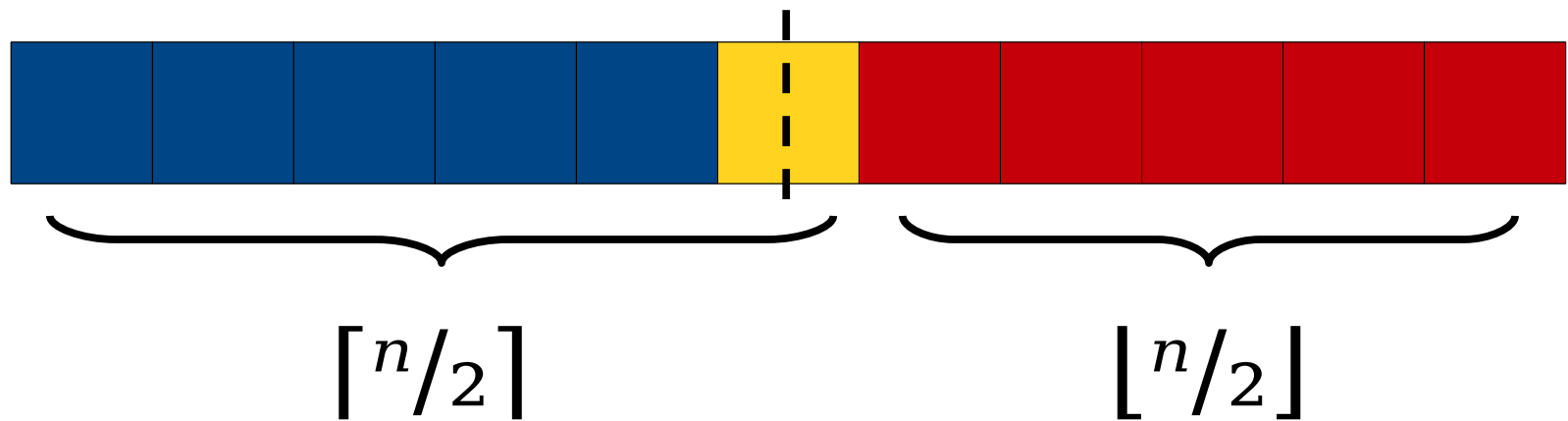


$$n = 2k$$

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# Let's Draw Some Pictures!



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*Case 1:  $n$  is even.*

This is called a *proof by cases* (or *proof by exhaustion*). We split apart into one or more cases and confirm that the result is indeed true in each of them.

*Case 2:  $n$  is odd.*

(Think of it like an if/else or switch statement.)

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In either case, we see that  $\lfloor n/2 \rfloor + \lceil n/2 \rceil = n$ , as required.  $\square$

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At the end of a split into cases, it's a nice courtesy to explain to the reader what it was that you established in each case.

$$= n.$$

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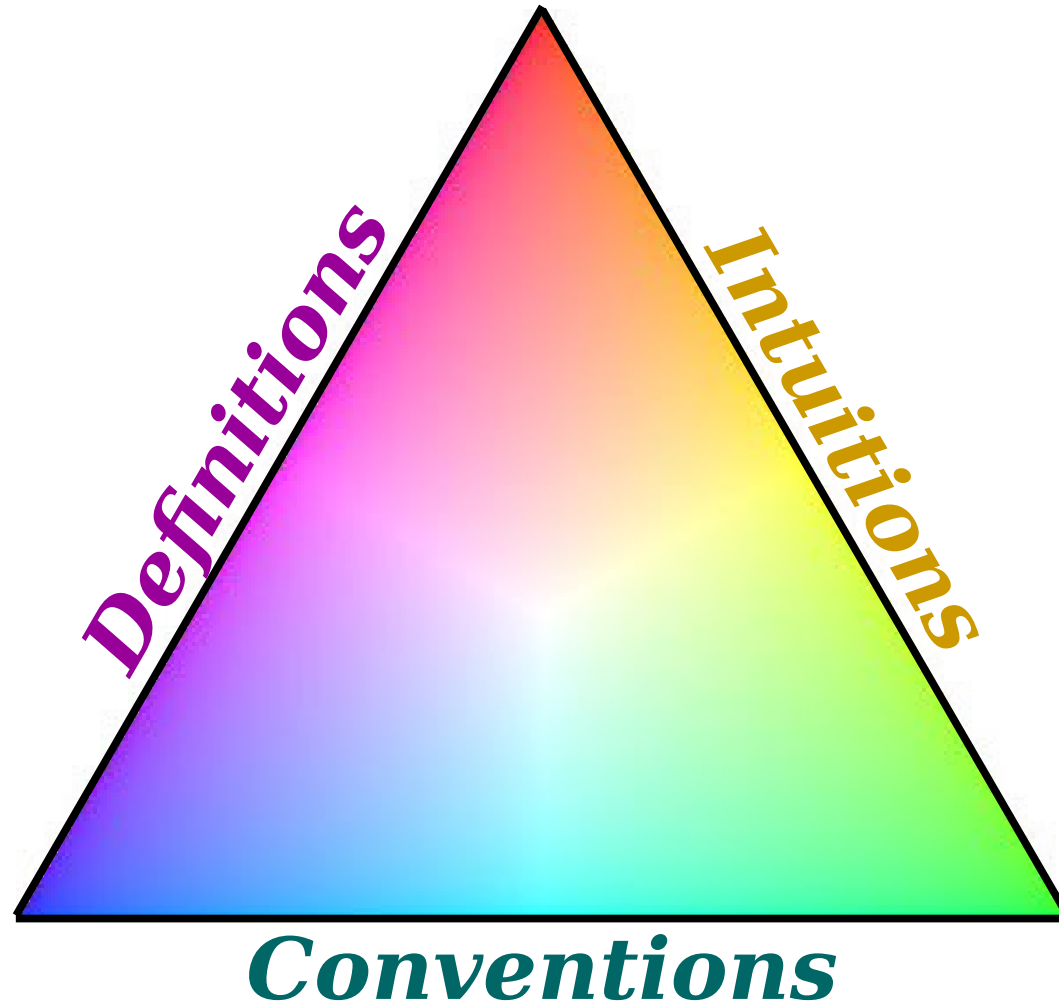
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To Recap



---

Writing a good proof requires a blend of definitions, intuitions, and conventions.

An integer  $n$  is **even** if there is an integer  $k$  where  $n = 2k$ .

An integer  $n$  is **odd** if there is an integer  $k$  where  $n = 2k+1$ .

---

Definitions tell us what we need to do in a proof. Many proofs directly reference these definitions.

**Let's Draw Some Pictures!**

**Let's Do Some Math!**

**Let's Try Some Examples!**

---

Building intuition for results requires creativity,  
trial, and error.

- Prove universal statements by making arbitrary choices.
- Prove existential statements by making concrete choices.
- Prove “If  $P$ , then  $Q$ ” by assuming  $P$  and proving  $Q$ .
- Write in complete sentences.
- Number sub-formulas when referring to them.
- Summarize what was shown in proofs by cases.
- Articulate your start and end points.

---

Mathematical proofs have established conventions that increase rigor and readability.

# Your Action Items

- ***Read “Guide to  $\in$  and  $\subseteq$ ,” “Guide to Proofs,” and “Guide to Partners.”***
  - These will be very useful for PS1.
- ***Finish and submit Problem Set 0.***
  - Don't put this off until the last minute!
- ***Watch Lecture 02.***
  - Curl up near a warm fire and enjoy learning some more proof magic!
- ***Start Problem Set 1.***
  - At a minimum, read over the problems so you know what's being asked.

# Next Time (Virtually!)

- ***Indirect Proofs***

- How do you prove something without actually proving it?

- ***Mathematical Implications***

- What exactly does “if  $P$ , then  $Q$ ” mean?

- ***Proof by Contrapositive***

- A helpful technique for proving implications.

- ***Proof by Contradiction***

- Proving something is true by showing it can't be false.

## Appendix: *Proofs as Dialogs*

# Proofs as a Dialog

Let  $n$  be an arbitrary odd integer.

Since  $n$  is an odd integer, there is an integer  $k$  such that  $n = 2k + 1$ .

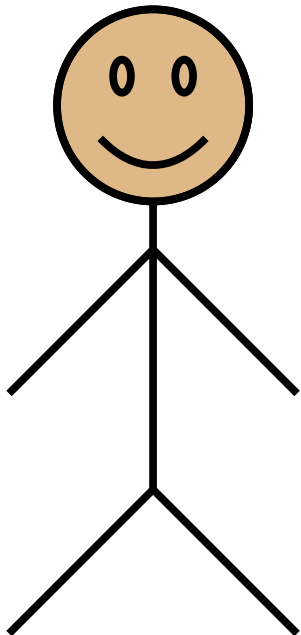
Now, let  $z = k - 34$ .

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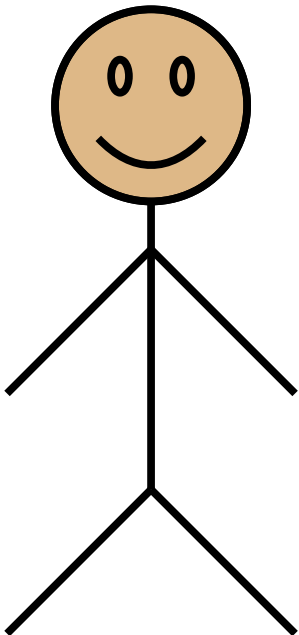
***Proof Writer (You)***

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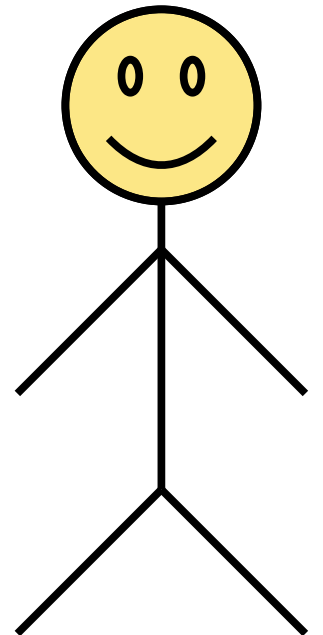
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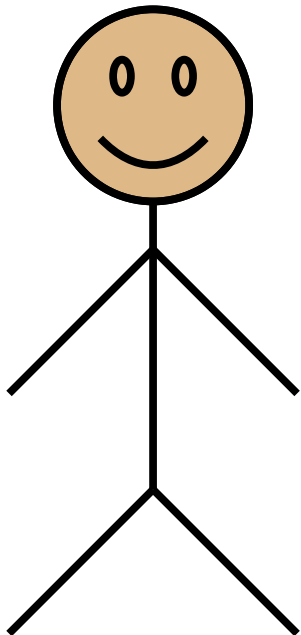
*Proof Reader*

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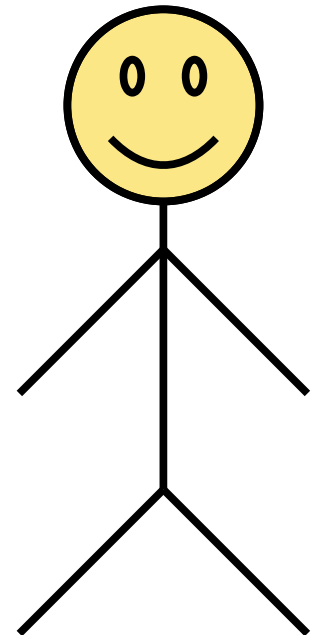
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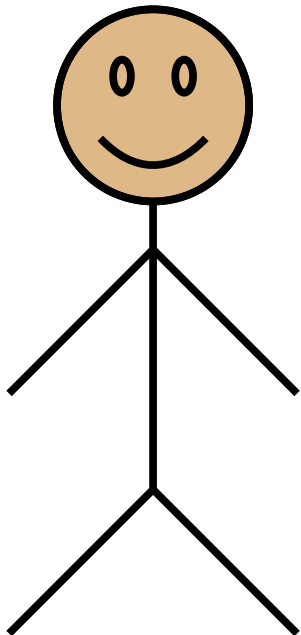
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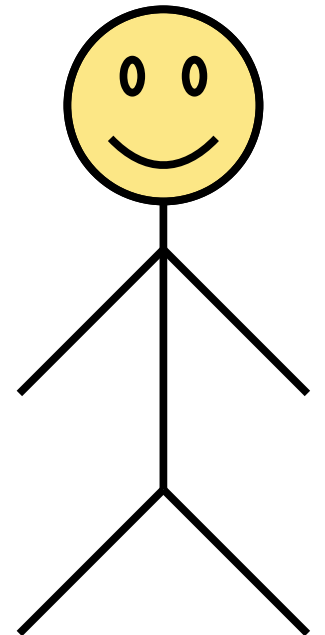
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$$n = 137$$

*Reader Picks*



*Proof Writer (You)*



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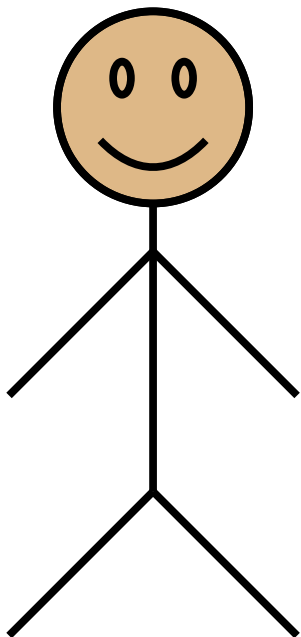
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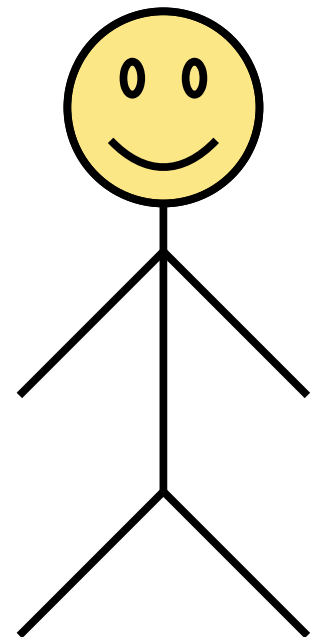
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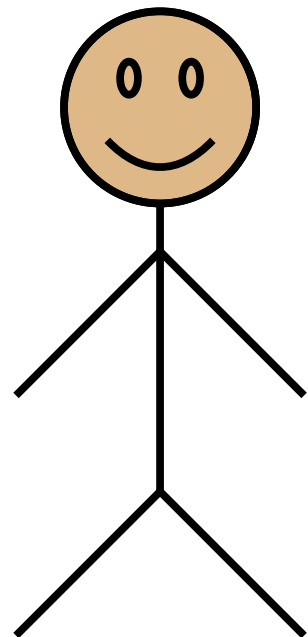
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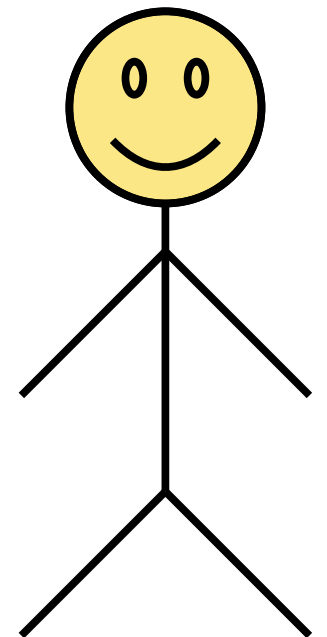
***Proof Writer (You)***

$k = 68$

***Neither Picks***

$n = 137$

***Reader Picks***



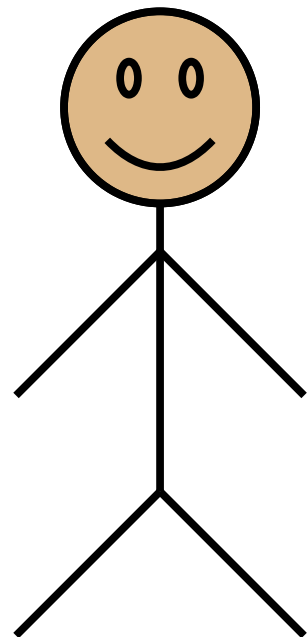
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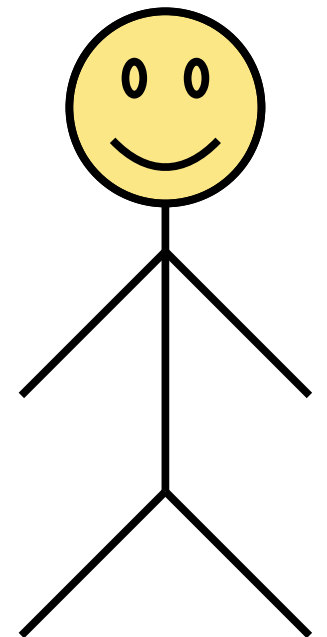
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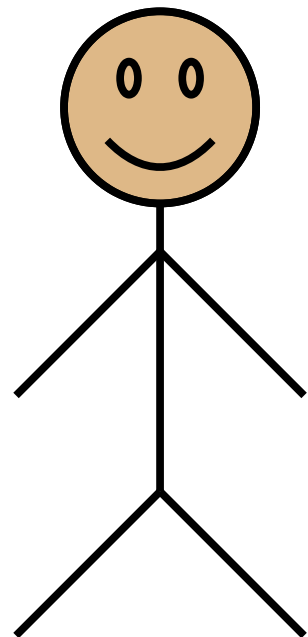
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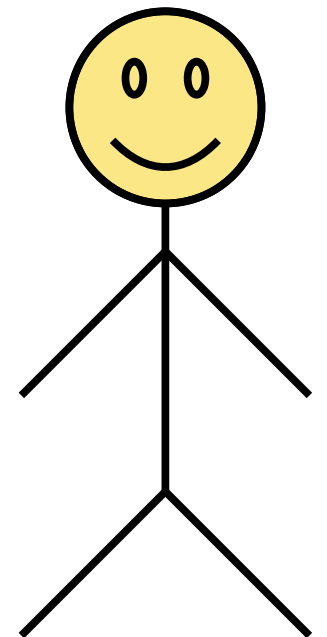
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$$n = 137$$

***Reader Picks***



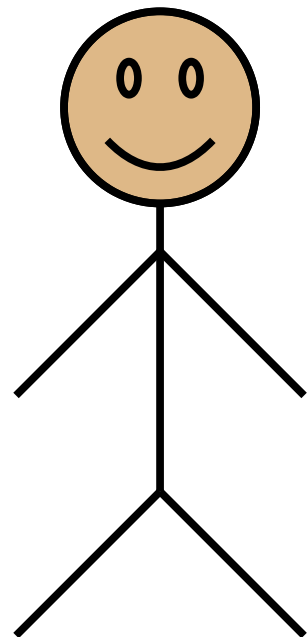
***Proof Reader***

# Proofs as a Dialog

Let  $n$  be an arbitrary odd integer.

Since  $n$  is an odd integer, there is an integer  $k$  such that  $n = 2k + 1$ .

Now, let  $z = k - 34$ .



*Proof Writer (You)*

$z = 34$

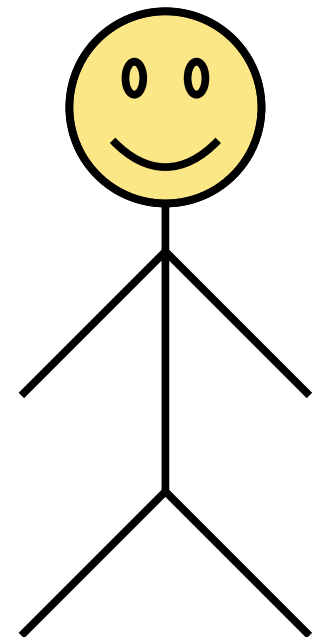
*Writer Picks*

$k = 68$

*Neither Picks*

$n = 137$

*Reader Picks*



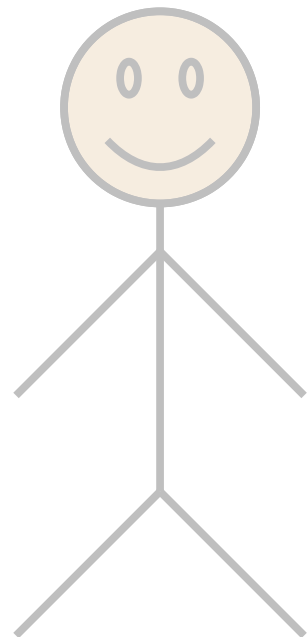
*Proof Reader*

# Proofs as a Dialog

Let  $n$  be an arbitrary odd integer.

Since  $n$  is an odd integer, there is an integer  $k$  such that  $n = 2k + 1$ .

Now, let  $z = k - 34$ .



*Proof Writer (You)*

$$z = 34$$

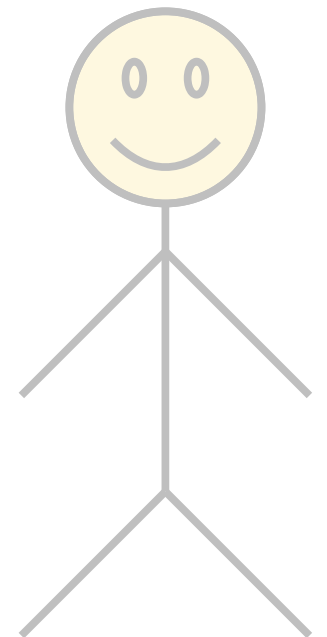
*Writer Picks*

$$k = 68$$

*Neither Picks*

$$n = 137$$

*Reader Picks*

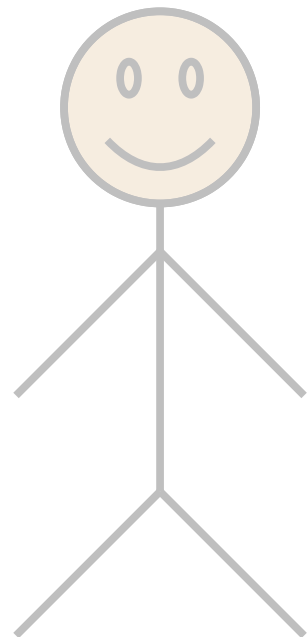


*Proof Reader*

Each of these variables has a distinct, assigned value.

Since Each variable was either picked by the reader, picked by the writer, or has a value that can be determined from other variables.

Now, let  $z = k - 34$ .



*Proof Writer (You)*

$z = 34$

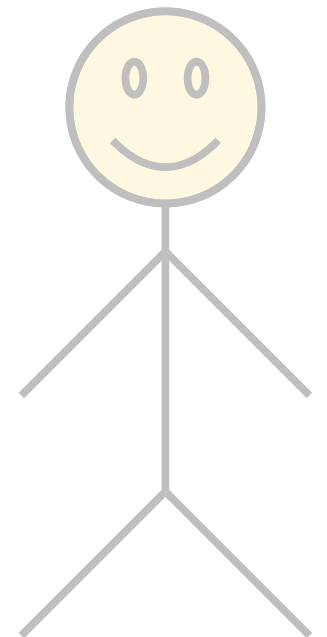
*Writer Picks*

$k = 68$

*Neither Picks*

$n = 137$

*Reader Picks*



*Proof Reader*

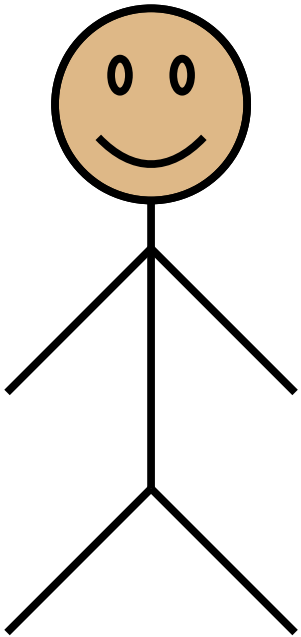
# Who Owns What?

- The **reader** chooses and owns a value if you use wording like this:
  - Pick a natural number  $n$ .
  - Consider some  $n \in \mathbb{N}$ .
  - Fix a natural number  $n$ .
  - Let  $n$  be a natural number.
- The **writer** (you) chooses and owns a value if you use wording like this:
  - Let  $r = n + 1$ .
  - Pick  $s = n$ .
- **Neither** of you chooses a value if you use wording like this:
  - Since  $n$  is even, we know there is some  $k \in \mathbb{Z}$  where  $n = 2k$ .
  - Because  $n$  is odd, there must be some integer  $k$  where  $n = 2k + 1$ .

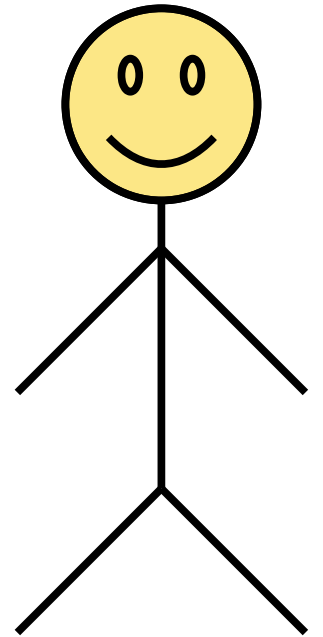
# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

Then for any even  $x$ , we know that  $x+1$  is odd.



*Proof Writer (You)*

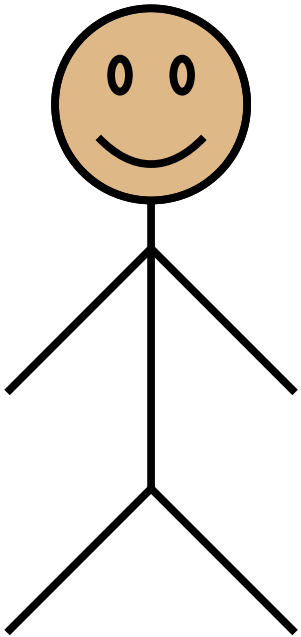


*Proof Reader*

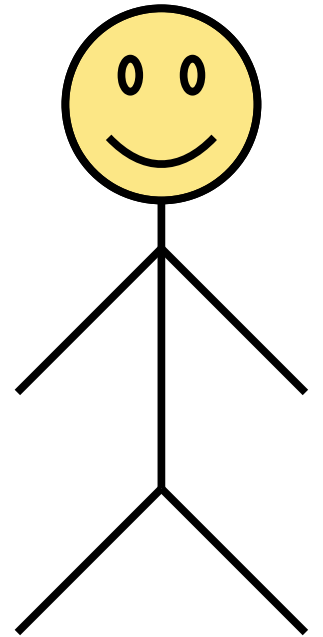
# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

Then for any even  $x$ , we know that  $x+1$  is odd.



*Proof Writer (You)*

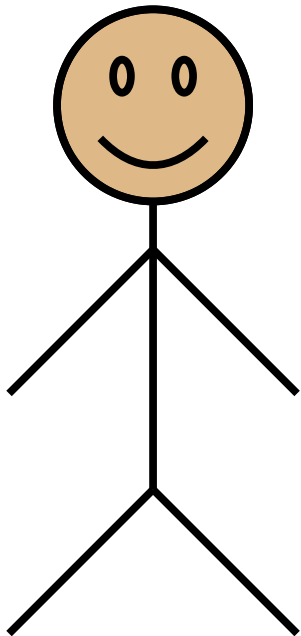


*Proof Reader*

# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

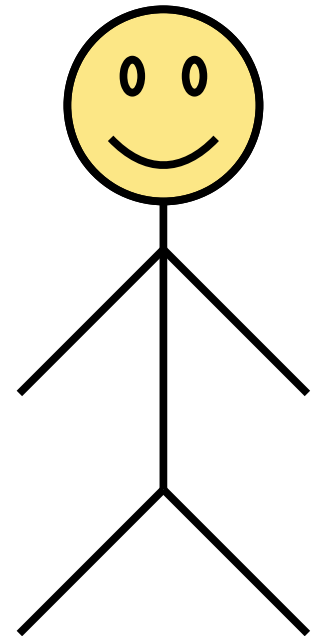
Then for any even  $x$ , we know that  $x+1$  is odd.



***Proof Writer (You)***

$$x = 242$$

***Reader Picks***

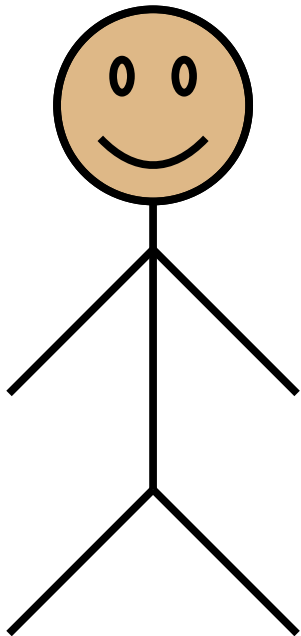


***Proof Reader***

# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

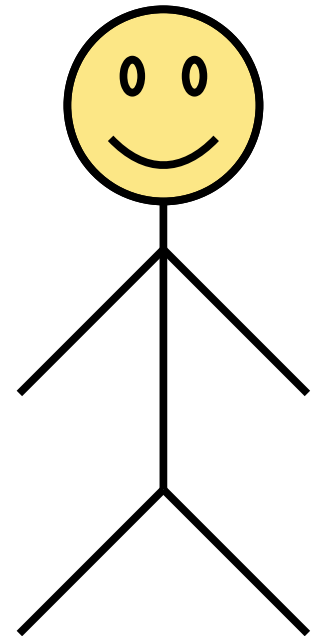
Then for any even  $x$ , we know that  $x+1$  is odd.



***Proof Writer (You)***

$$x = 242$$

***Reader Picks***

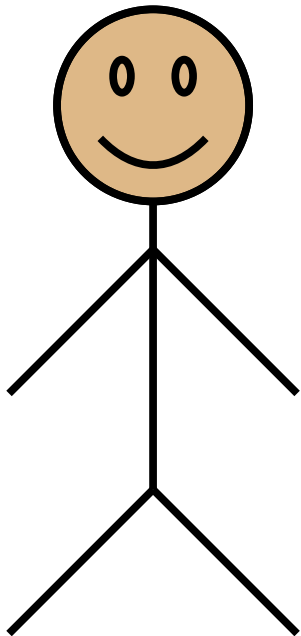


***Proof Reader***

# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

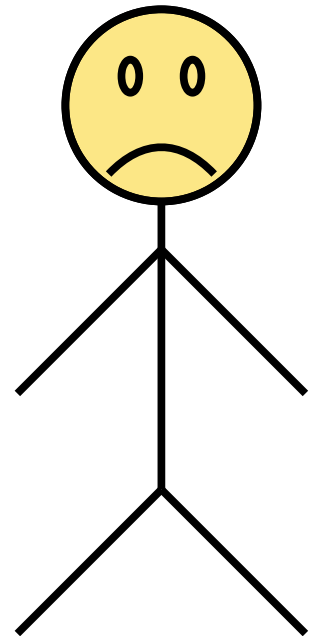
Then for any even  $x$ , we know that  $x+1$  is odd.



*Proof Writer (You)*

$$x = 242$$

*Reader Picks*

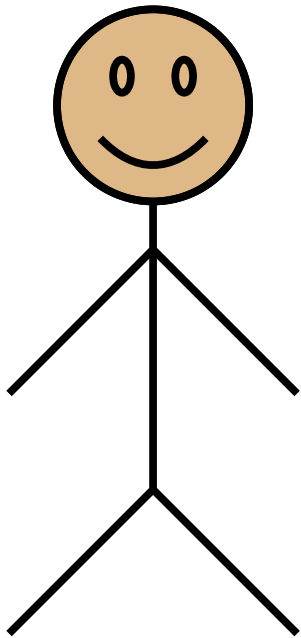


*Proof Reader*

# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

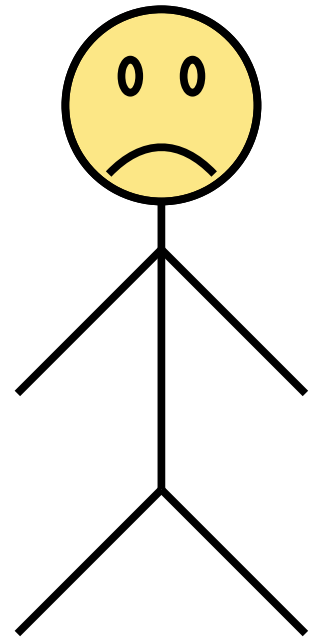
Then **for any even  $x$** , we know that  $x+1$  is odd.



***Proof Writer (You)***

$$x = 242$$

***Reader Picks***

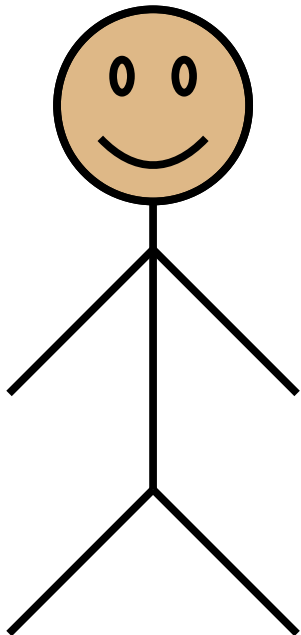


***Proof Reader***

# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

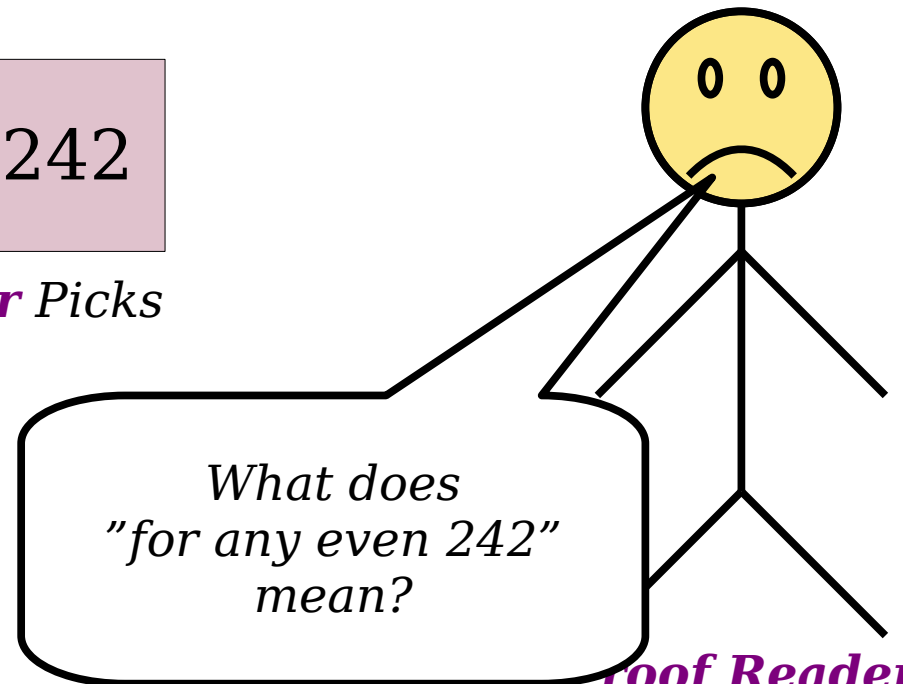
Then **for any even  $x$** , we know that  $x+1$  is odd.



**Proof Writer (You)**

$$x = 242$$

**Reader Picks**

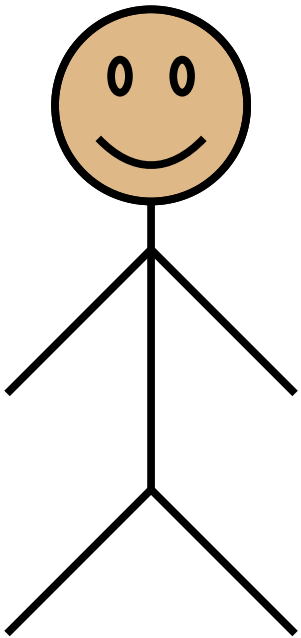


**Proof Reader**

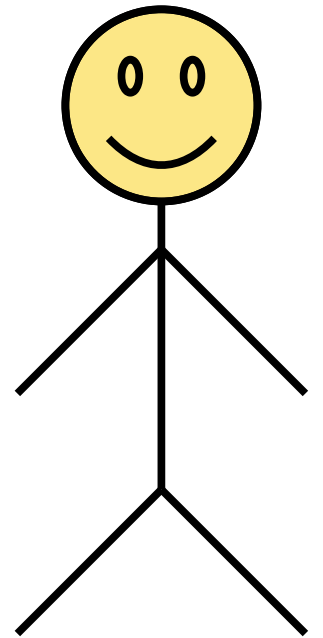
# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

Since  $x$  is even, we know that  $x+1$  is odd.



*Proof Writer (You)*

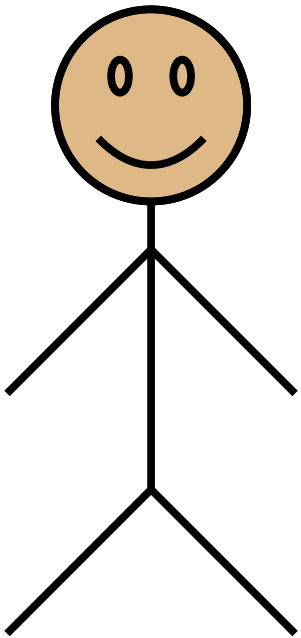


*Proof Reader*

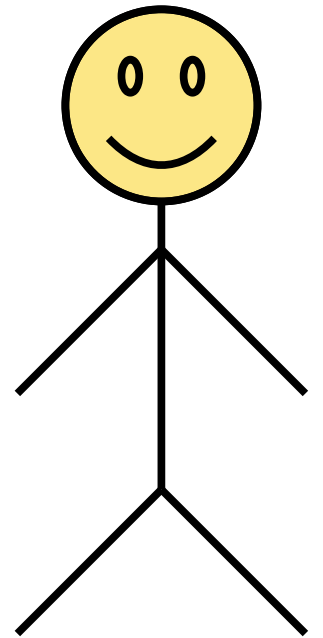
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Let  $x$  be an arbitrary even integer.

Since  $x$  is even, we know that  $x+1$  is odd.



***Proof Writer (You)***

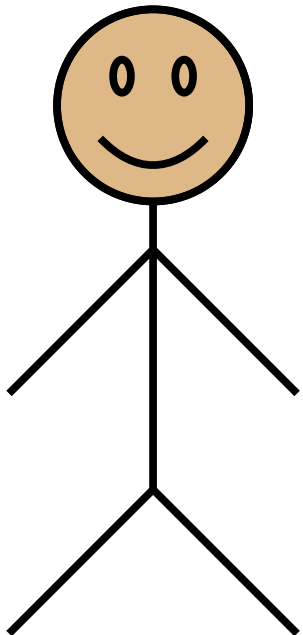


***Proof Reader***

# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

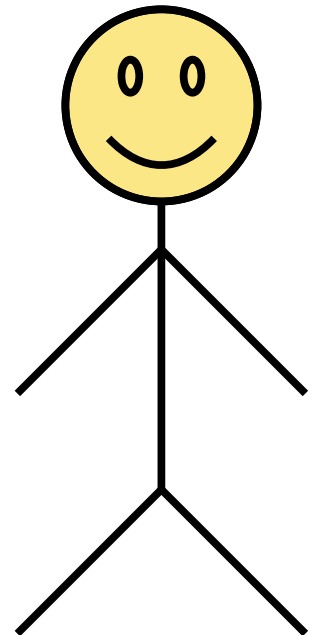
Since  $x$  is even, we know that  $x+1$  is odd.



***Proof Writer (You)***

$$x = 242$$

***Reader Picks***

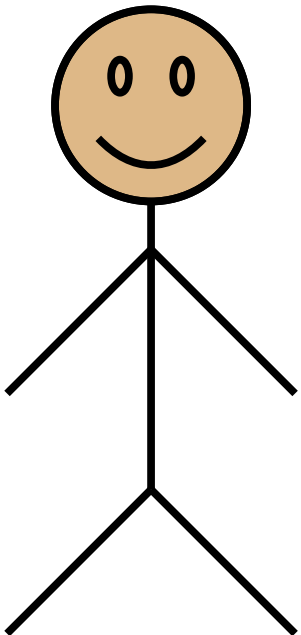


***Proof Reader***

# Proofs as a Dialog

Let  $x$  be an arbitrary even integer.

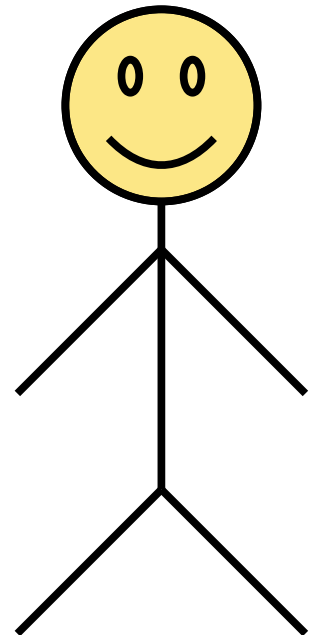
Since  $x$  is even, we know that  $x+1$  is odd.



*Proof Writer (You)*

$$x = 242$$

*Reader Picks*



*Proof Reader*

***Every variable needs a value.***

***Avoid talking about “all  $x$ ” or “every  $x$ ”  
when manipulating something  
concrete.***

***To prove something is true for any  
choice of a value for  $x$ , let the reader  
pick  $x$ .***

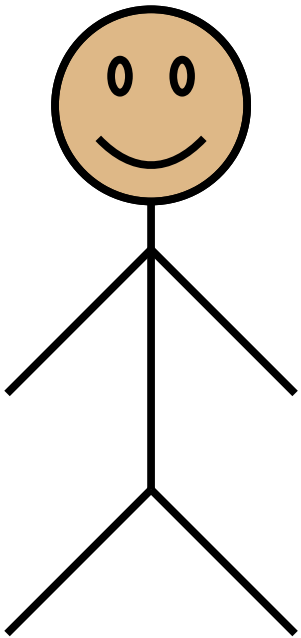
***Once you've said something like***

Let  $x$  be an integer.  
Consider an arbitrary  $x \in \mathbb{Z}$ .  
Pick any  $x$ .

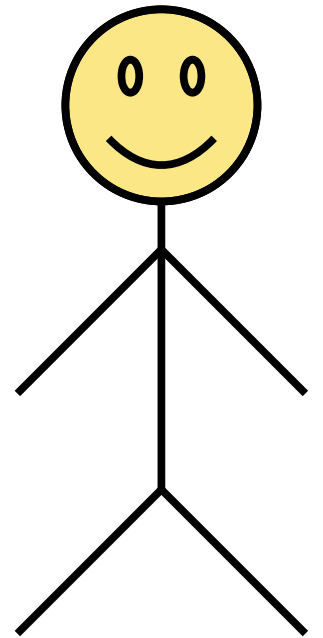
***Do not say things like the following:***

This means that ***for any***  $x \in \mathbb{Z} \dots$   
So ***for all***  $x \in \mathbb{Z} \dots$

# Proofs as a Dialog



*Proof Writer (You)*

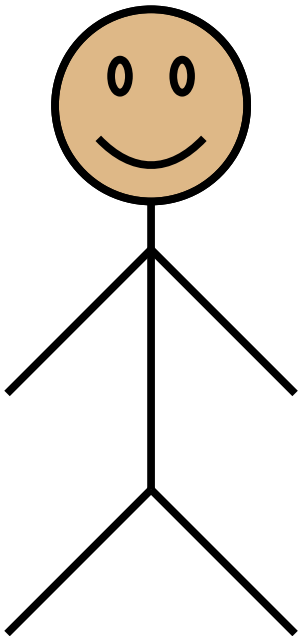


*Proof Reader*

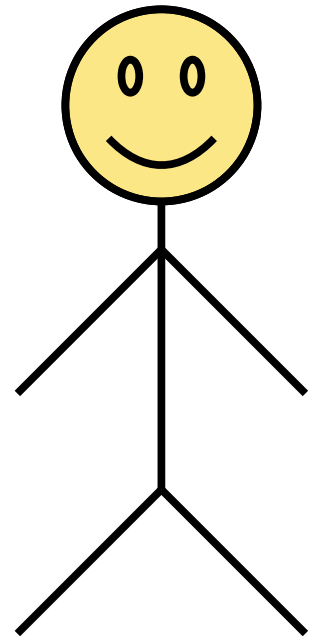
# Proofs as a Dialog

Pick two integers  $m$  and  $n$  where  $m+n$  is odd.

Let  $n = 1$ , which means that  $m+1$  is odd.



***Proof Writer (You)***

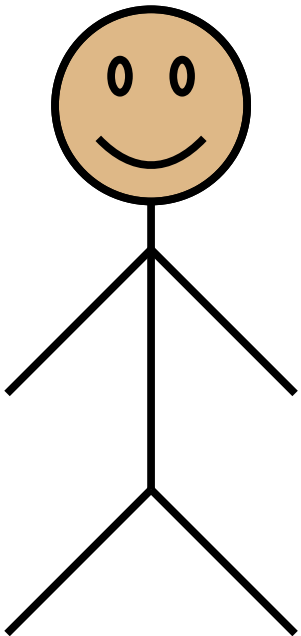


***Proof Reader***

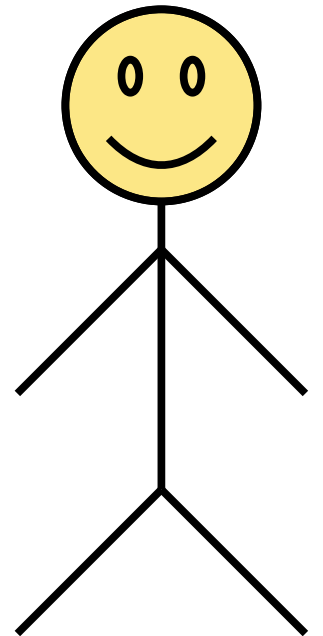
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Pick two integers  $m$  and  $n$  where  $m+n$  is odd.

Let  $n = 1$ , which means that  $m+1$  is odd.



***Proof Writer (You)***

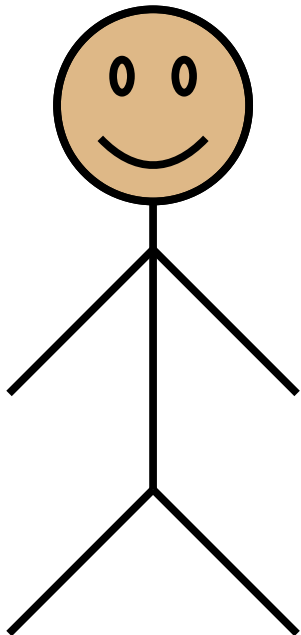


***Proof Reader***

# Proofs as a Dialog

Pick two integers  $m$  and  $n$  where  $m+n$  is odd.

Let  $n = 1$ , which means that  $m+1$  is odd.



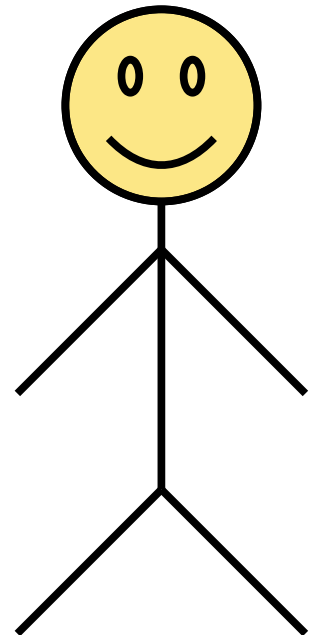
***Proof Writer (You)***

$$m = 103$$

***Reader Picks***

$$n = 166$$

***Reader Picks***

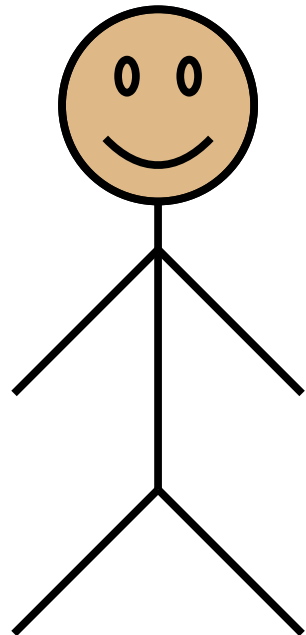


***Proof Reader***

# Proofs as a Dialog

Pick two integers  $m$  and  $n$  where  $m+n$  is odd.

Let  $n = 1$ , which means that  $m+1$  is odd.



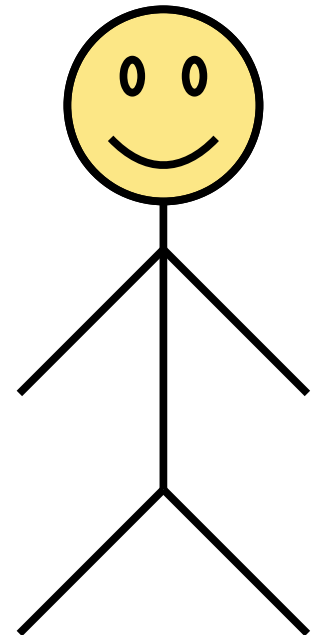
***Proof Writer (You)***

$$m = 103$$

***Reader Picks***

$$n = 166$$

***Reader Picks***

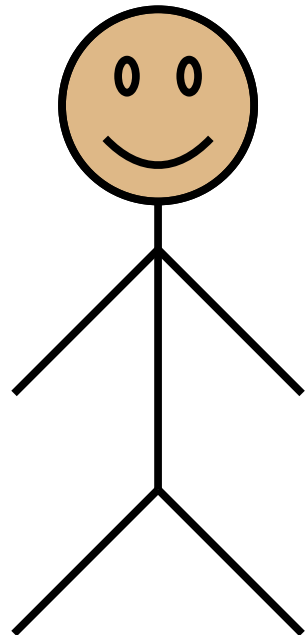


***Proof Reader***

# Proofs as a Dialog

Pick two integers  $m$  and  $n$  where  $m+n$  is odd.

Let  $n = 1$ , which means that  $m+1$  is odd.



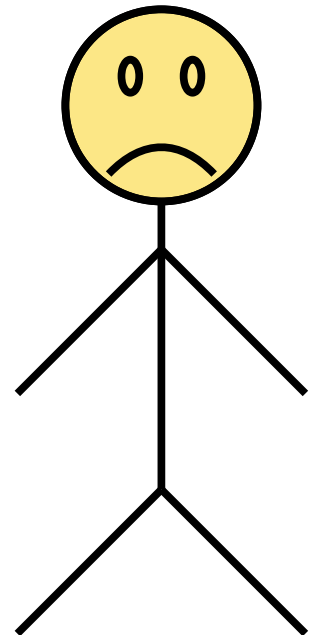
*Proof Writer (You)*

$$m = 103$$

*Reader Picks*

$$n = 166$$

*Reader Picks*

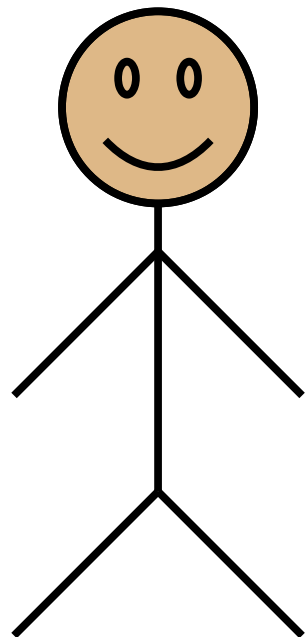


*Proof Reader*

# Proofs as a Dialog

Pick two integers  $m$  and  $n$  where  $m+n$  is odd.

Let  $n = 1$ , which means that  $m+1$  is odd.



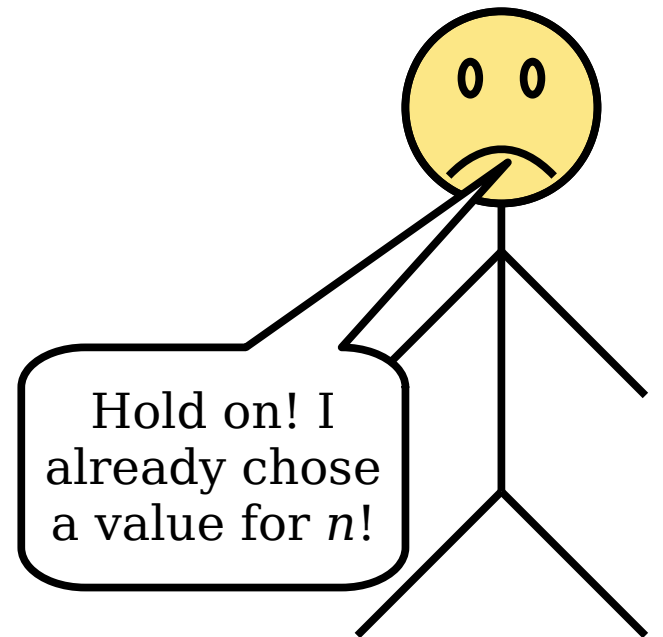
***Proof Writer (You)***

$$m = 103$$

***Reader Picks***

$$n = 166$$

***Reader Picks***



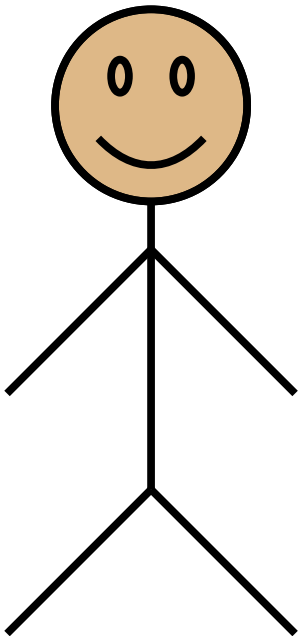
Hold on! I already chose a value for  $n$ !

***Proof Reader***

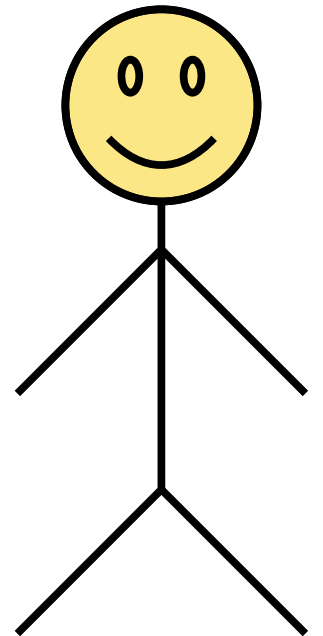
# Proofs as a Dialog

Let  $n = 1$ .

Pick any integer  $m$  where  $m+1$  is odd.



*Proof Writer (You)*

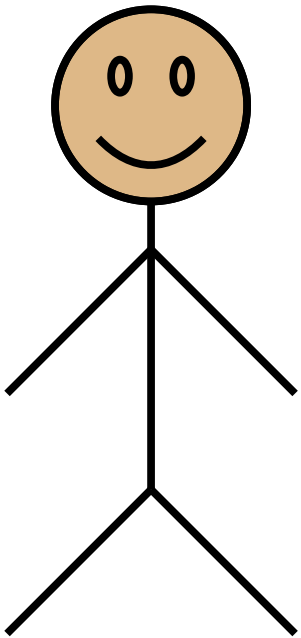


*Proof Reader*

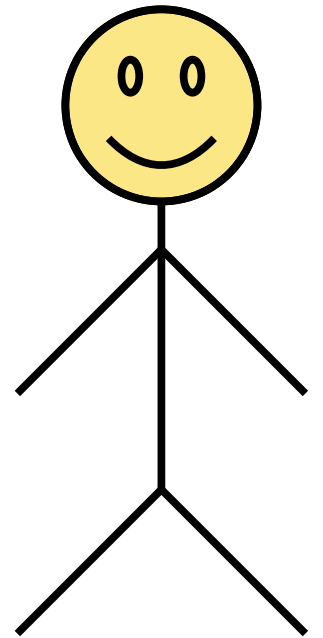
# Proofs as a Dialog

Let  $n = 1$ .

Pick any integer  $m$  where  $m+1$  is odd.



***Proof Writer (You)***

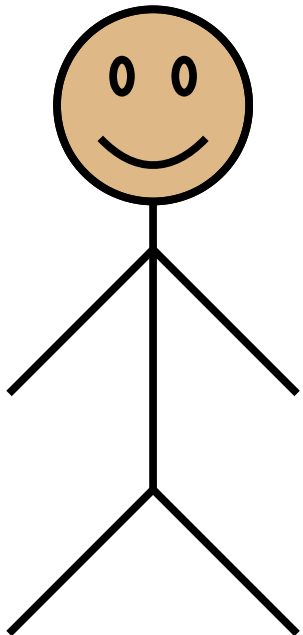


***Proof Reader***

# Proofs as a Dialog

Let  $n = 1$ .

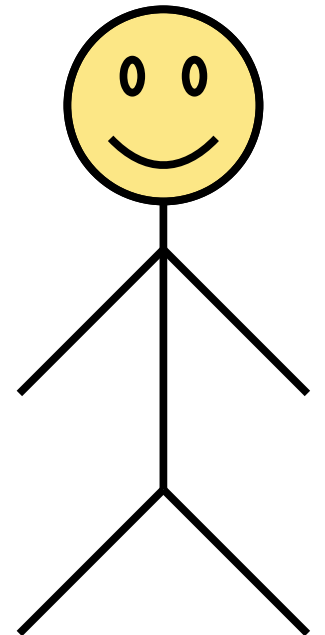
Pick any integer  $m$  where  $m+1$  is odd.



***Proof Writer (You)***

$n = 1$

***Writer Picks***

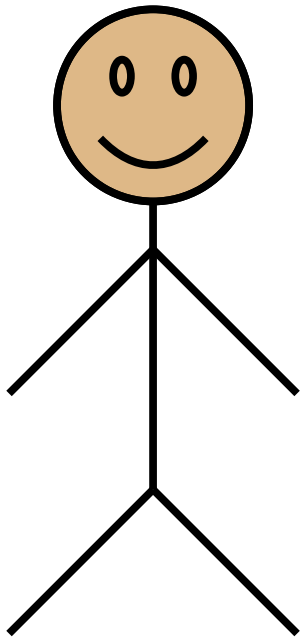


***Proof Reader***

# Proofs as a Dialog

Let  $n = 1$ .

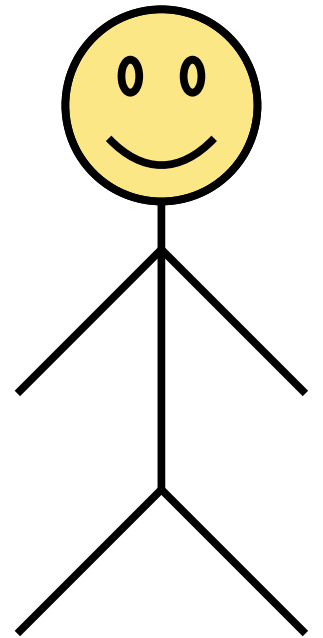
Pick any integer  $m$  where  $m+1$  is odd.



***Proof Writer (You)***

$n = 1$

***Writer Picks***

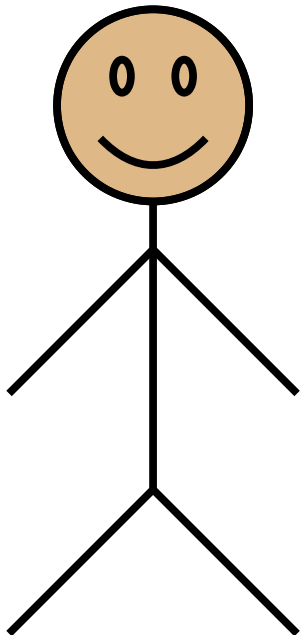


***Proof Reader***

# Proofs as a Dialog

Let  $n = 1$ .

Pick any integer  $m$  where  $m+1$  is odd.



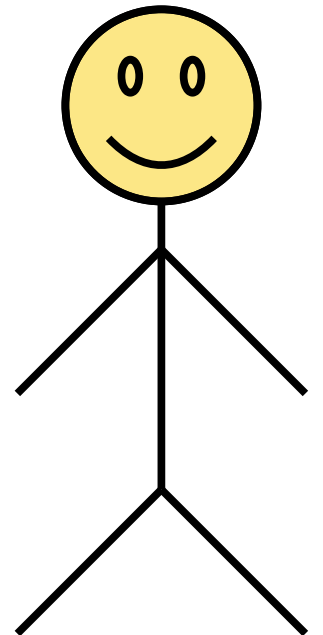
***Proof Writer (You)***

$$m = 166$$

***Reader Picks***

$$n = 1$$

***Writer Picks***

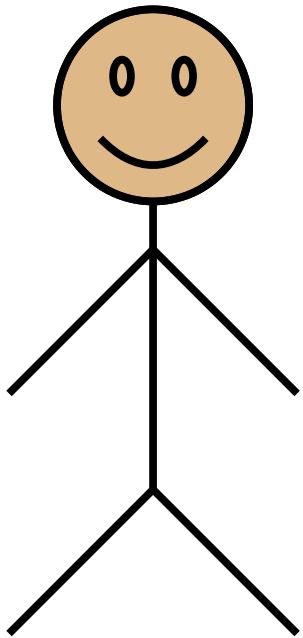


***Proof Reader***

# Proofs as a Dialog

Let  $n = 1$ .

Pick any integer  $m$  where  $m+1$  is odd.



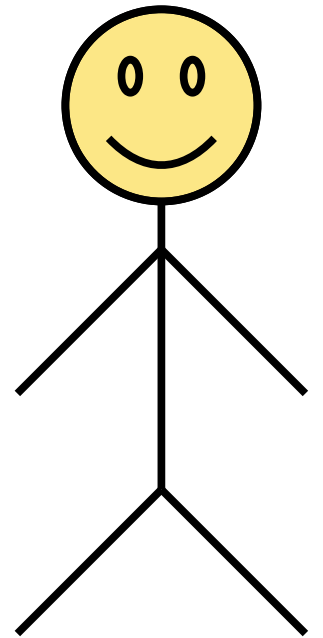
***Proof Writer (You)***

$$m = 166$$

***Reader Picks***

$$n = 1$$

***Writer Picks***



***Proof Reader***

# Proofs as a Dialog

Do we even need  $n$  here?

Let  $n = 1$ .

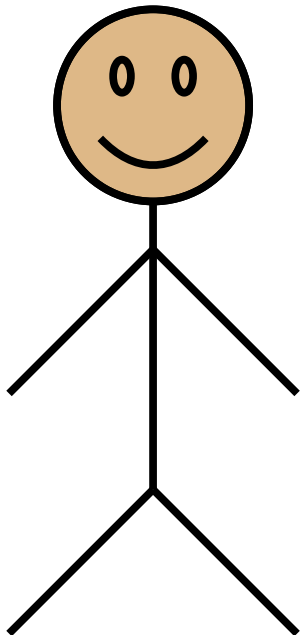
Pick any integer  $m$  where  $m+1$  is odd.

$m = 166$

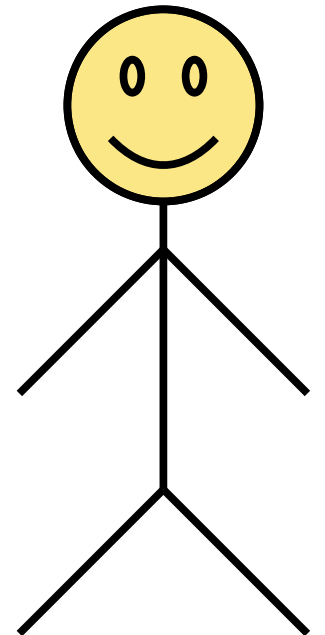
*Reader Picks*

$n = 1$

*Writer Picks*



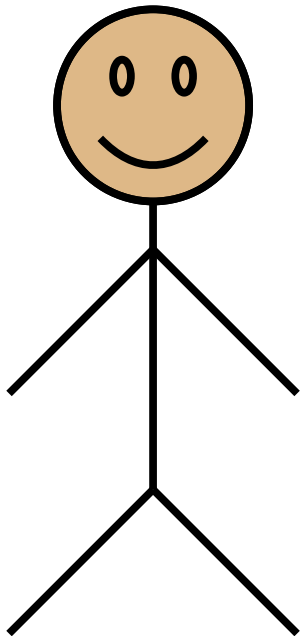
*Proof Writer (You)*



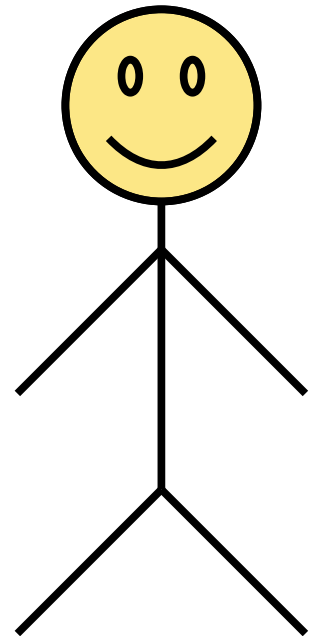
*Proof Reader*

# Proofs as a Dialog

Pick any integer  $m$  where  $m+1$  is odd.



***Proof Writer (You)***



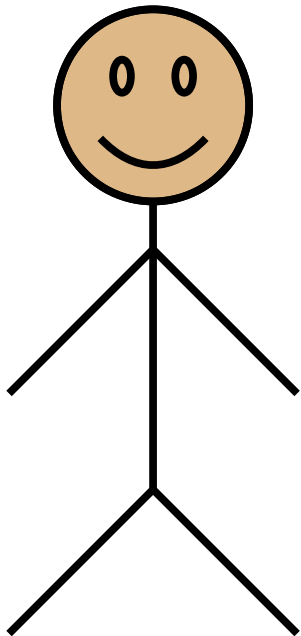
***Proof Reader***

# Proofs as a Dialog

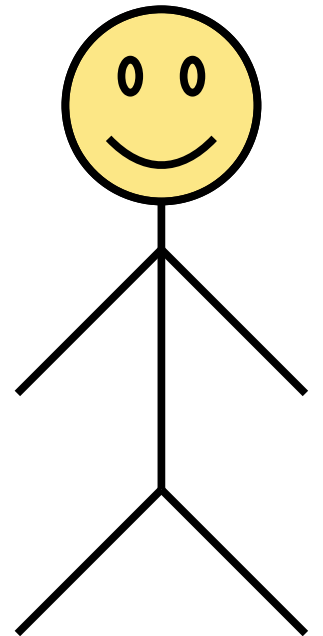
Pick any integer  $m$  where  $m+1$  is odd.

$$m = 166$$

*Reader Picks*



*Proof Writer (You)*



*Proof Reader*

***Be mindful of who owns what variable.***

***Don't change something you don't own.***

***You don't always need to name things,  
especially if they already have a name.***